International R&D Spillovers and Asset Prices

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Abstract

We provide new empirical evidence of a relationship between asset prices and trade-induced international R&D spillovers; in particular, we find that pairs of countries that share more research and development though international trade exhibit more highly correlated stock market returns and less volatile exchange rates. We develop an endogenous growth model of innovation and international technology diffusion that accounts for these novel empirical findings. A calibrated version of the model matches several important asset-pricing and quantity moments, alleviating some of the classical quantity–price puzzles highlighted in the literature on international macroeconomics.

Keywords: innovation, international diffusion, trade, asset prices, recursive preferences

JEL classification: F3, F4, O3

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1 Introduction

Technological innovation is a fundamental source of sustained economic growth (Romer 1990). Asset prices reflect changes in an economy's future growth prospects and thus capture variations in technological innovation (Comin, Gertler, and Santacreu 2009 and Kung and Schmid 2015). In an international setting, technology diffuses across countries. One channel for that mechanism has received particular attention in the literature: the trade in products (varieties) that incorporate such technology. Thus a country's growth rate depends not only on its own innovation but also on the innovation efforts of its trading partners (Coe and Helpman 1995; Keller 1998). It follows that the dynamics of technological innovation–both within and across countries–may inform us about the comovements of international asset returns.[1]

In this paper, we investigate the link between international trade—our measure of how technological innovation diffuses across countries—and comovements in asset prices. Our first contribution is empirical. Using bilateral trade data in conjunction with data on investment in research and development (R&D), we document the following empirical regularities: country pairs that share R&D by trading more with each other exhibit *greater* correlation in stock market returns as well as *less* volatile exchange rates. These patterns are robust to controlling for alternative measures of R&D and trade across each country pair, which suggests that international R&D spillovers play an important role in capturing how the international diffusion of technological innovation is related to asset prices.

Our next contribution is theoretical. In particular, we build a two-country endogenous growth model of innovation and international technology diffusion through trade in varieties that rationalizes our empirical findings. Growth in each country is driven by the accumulation of technology through endogenous innovation. We posit that the technology embodied in intermediate goods spreads across countries via international trade. As a result, the aggregate expected productivity of a country depends on its own innovation *and* on the foreign innovations embodied in imported intermediate products. Preferences are recursive, so that consumers care about the timing of resolution of uncertainty and fear variation in the econ-

[1]The international trade literature has also argued in favor of trade in varieties as the channel through which R&D diffuses across countries (Broda, Greenfield, and Weinstein 2006; Bøler, Moxnes, and Ulltveit-Moe 2012; Santacreu 2015). Other channels include multinational firms spreading the benefits of R&D across countries (Ramondo 2009; Guadalupe, Kuzmina, and Thomas 2012; Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple 2015; Cravino and Levchenko 2015) and knowledge spillovers by way of international networks (Cai and Li 2012). The analysis of these latter channels is beyond the scope of our paper.

omy's long-run future growth prospects. We assume that international financial markets are complete. Endogenous innovation, together with recursive preferences, makes the equilibrium growth path risky due to its effect on the present discounted value of future profits of all firms in the economy.

Our endogenous growth mechanism works as follows. R&D drives a small and persistent component of equilibrium growth rates. International diffusion through trade in varieties renders this component common across countries. The intuition is that a technology shock in the home country affects the incentives to innovate in that country and also abroad, which in turn affects the prospects of global growth. Therefore, a short-run technology shock in the home country has a long-run effect on the dynamics of future growth rates in both the home and the foreign economy.

If preferences are recursive, the variations in the economy's future prospects will have a significant effect on asset prices. As technological innovation diffuses across countries through trade in varieties, it generates a sizable common component in discount rates that drives up the correlation in stock market returns while reducing the volatility of exchange rates. Our model replicates these international asset–pricing facts together with a realistic calibration of other macroeconomic quantities. More specifically, the model predicts—in line with the data— that realized consumption growth is only somewhat correlated across countries; the reason is that realized growth is driven mainly by exogenous technology shocks, which exhibit low levels of cross-country correlation.

We calibrate the model to match our novel empirical findings. Consistently with our predictions, we find that asset prices are largely driven by the economy's long-run future prospects, whereas international quantities are mostly driven by current technological levels. When growth is endogenous, stock market returns are highly correlated across countries—and even more so under increased international R&D spillovers. Furthermore, the exchange rate in our model is as volatile as in the data, and its volatility decreases when international R&D spillovers increase. Importantly, our model delivers sensible dynamics for consumption growth, Sharpe ratios, and risk-free rates across countries.

Our paper is related to several strands of the literature. First, the macroeconomic mechanism is related to the literature on endogenous growth through innovation. In our model, technological progress increases with the number of intermediate goods that incorporate technology. Kung and Schmid (2015) extend Romer (1990) to include recursive preferences.

We develop our model along those lines and extend it to an international setting so that it accounts for our novel empirical findings on the relations among trade, R&D, and asset prices.[2]

The second strand of the literature is related to papers that analyze the implications for asset pricing of the canonical “real business cycle model”. Examples include habit-based models (Jermann 1998; Boldrin, Christiano, and Fisher 2001), studies on recursive preferences (Tallarini 2000; Campanale, Castro, and Clementi 2010; Kuehn 2007; Kaltenbrunner and Lochstoer 2010; and Papanikolaou 2011), and disaster risk analysis (Gourio 2012). Our paper differs from these by focusing on the international dimension of asset prices.

The third related strand of literature is that on technology adoption and innovation through international trade in varieties (e.g., Broda, Greenfield, and Weinstein 2006; Santacreu 2015). Using highly disaggregated trade data, this research finds that adopting foreign innovations–by trade in varieties–affects the home country's growth rate. However, these authors do not discuss the asset–pricing implications of their mechanism, which is one of the main contributions of our study.

Finally, our paper is related to the literature on asset pricing with long-run risk that began with the seminal one-country model of Bansal and Yaron (2004) and was then applied to the international setting by Bansal and Shaliastovich (2013), Colacito and Croce (2011), and Colacito and Croce (2013). Whereas these papers specify global long-run risk exogenously, our model shows how such risk—which is highly persistent within countries and highly correlated across countries—arises endogenously through innovation and the international diffusion of R&D. From a methodological standpoint, our paper is related to that of Croce, Nguyen, and Schmid (2013), who examine the role of fiscal policy in an international endogenous growth model featuring agents who are concerned about robustness. Unlike that paper, ours focuses on the empirical links among trade, R&D, and asset prices in the context of an endogenous growth model with recursive preferences.

The rest of our paper is organized as follows. Section 2 reports our main empirical findings, and Section 3 describes the model. In Section 4 we describe the main mechanism at work, and Section 5 presents the calibration and our quantitative results. Section 6 concludes.

[2] Several papers have examined the link between technological growth and prices; examples include Pastor and Veronesi (2009), Garleanu, Panageas, and Yu (2012), and Garleanu, Kogan, and Panageas (2012). These papers assume that technology growth is *exogenous*. In contrast, and consistently with our empirical findings, we focus on the relation between international asset prices and *endogenous* growth through R&D.

2 Innovation, Trade, and Asset Prices: Empirical Evidence

A striking feature of international macroeconomics data is the low cross-country correlation of consumption growth as compared to the correlation of stock market returns: for the average country, the former is 0.33 whereas the latter is more than twice as high, at about 0.77.[3] We also find that, for the period analyzed here, the annualized average volatility of the exchange rate in our sample countries is approximately 10%. Standard international macroeconomic models have been unable to reconcile these empirical asset–pricing and quantity moments. We argue that the international spillovers of R&D resulting from international trade may be a significant driver of the dynamics of international asset prices. To the extent that the technology created by investing in R&D is embodied in a particular good, movements of goods across borders may well help diffuse those technologies.

In this section we investigate the empirical relevance of our proposed mechanism. We collect data on asset prices, international trade, and R&D before proceeding in two steps. After examining the correlation between broad measures of asset pricing moments and a measure of the R&D content of international trade, we perform a regression analysis to investigate the drivers of this relationship. Our main data sources are as follows: for international trade, the UN COMTRADE; for R&D, the OECD Science and Technology Indicators (Business Enterprise R&D (BERD) as a % of GDP); and for asset prices, Global Financial Data and Ken French's website.[4]

Our main data set covers the 1996–2013 period for a sample of 20 countries.[5] The choice of countries and time period was determined based on the availability of data for asset pricing, trade, and R&D. Details on these sources–and on the construction of measures used in our analysis–are given in Appendix A.

[3] The average, based on quarterly data, is taken over a sample of 20 countries for the period 1996–2013. More specific details are given in Appendix A.

[4] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[5] The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. For exchange rates, we use the same period but eliminate all eurozone countries (save Germany), Denmark and Singapore, which have instituted a managed exchange rate versus the Euro and the US dollar.

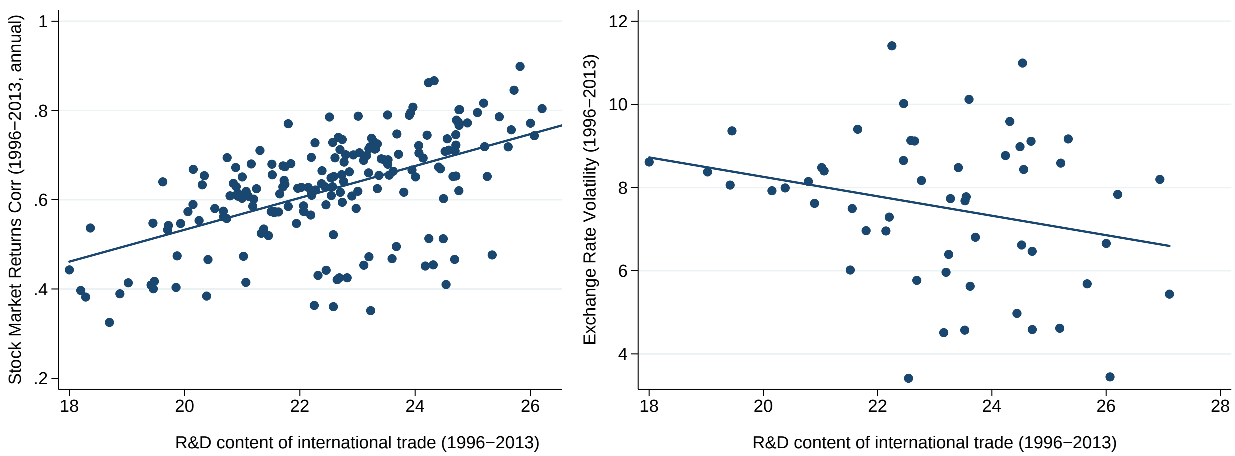

Figure 1: R&D content of trade and asset prices: time-series average for each country pair.

2.1 Correlation of Asset Pricing Moments, R&D, and International Trade

Here we analyze the correlations between broad measures of asset prices and measures of international trade and R&D. The goal is to determine whether the mechanism that we propose (i.e., R&D embodied in trade) plays any role in asset prices. We consider two statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the exchange rate. We start with monthly observations and construct annual, non-overlapping measures of cross-country correlation—for stock market returns—and volatility —for the exchange rate—per each ordered country pair.[6]

We then use data on R&D and international trade to construct a measure of the R&D content of trade. This is the key measure reflecting our mechanism of how R&D is shared across countries though international trade. For this purpose, we start by constructing the R&D stock for each country in our sample. Given a country's R&D intensity—defined as R&D expenditures over gross domestic product (GDP)—we apply the so-called perpetual inventory method using an annual depreciation rate of 15% (as in Coe and Helpman 1995; Nishioka and Ripoll 2012). Next, we follow the empirical literature and obtain a measure of the R&D content of international trade for a particular pair of countries i and j ($X_{i,j}^{\text{R\&D}}$), as

$$X_{i,j}^{\text{R\&D}} = \frac{X_{i,j}}{GDP_j} R\&D_j + \frac{X_{j,i}}{GDP_i} R\&D_i \tag{1}$$

[6]We also consider 60-month (5-year) overlapping measures with a 12-month overlap, and also the average across all periods. The results are virtually unchanged. See Appendix D.

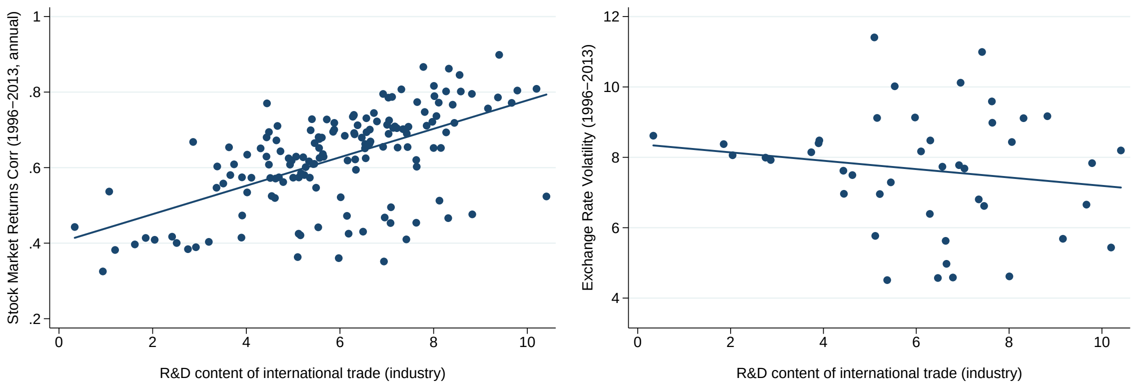

Figure 2: R&D content of trade and asset prices (industry measure): time-series average for each country pair.

where $X_{i,j}$ is total trade between exporter j and importer i, $R\&D_i$ is the stock of R&D in country i, and GDP_i is the gross domestic product of country i. This measure is constructed for each year in our sample.

We then asses the relationship (if any) between our measures of asset prices and the R&D content of international trade. Figure 1 reveals that pairs of countries whose trade is characterized by greater R&D content exhibit more highly correlated stock market returns and less volatile exchange rates. In the figure's two graphs, each solid circle represents the time-series average over the 1996-2013 period of each country pair in our sample. We have 380 unique country pairs with regard to the stock market correlation (left panel) and 108 unique country pairs with regard to the exchange rate volatility (right panel); recall that in the latter case, we omit Singapore, Denmark and all eurozone countries other than Germany.

The R&D content of international trade is computed using equation (1), and it may be subject to measurement error. Another problem with this measure is that pair of countries engaging in substantial trade with each other will exhibit a large R&D content of trade, even if they are not very innovative. One way to control for this limitation of our aggregate measure, is to employ trade and R&D data that are more disaggregated. We conduct a robustness analysis in which a measure of R&D knowledge embodied in intermediate inputs is calculated at the industry level (following closely Nishioka and Ripoll 2012). We use their global input-output matrix of 13 manufacturing industries and 32 countries for the years 1995, 2000, and 2005. Our more aggregated measure has a longer time series but fails to account for the disaggregation of R&D expenditures at the industry level. We then evaluate the extent to which this new measure correlates with our moments of asset prices. As Figure 2 shows, the results are broadly in line with those obtained using more aggregated measures. Country pairs that share more R&D through trade, using the input-output structure of the data, have more correlated stock market returns and less volatile exchange rates.

Taken together, these graphs suggest that R&D and international trade can inform us on the international dynamics of asset prices.

2.2 Regression Analysis

We now undertake a formal regression analysis in order to address the economic and statistical significance of the results presented in Section 2.1. Thus, we regress the asset-pricing moments of interest—namely, the cross-country correlation of stock market returns and the volatility of exchange rate—on our measure the R&D embodied in trade. To preserve the data's time-series aspect, we report the regression results for unique country pairs over the entire 1996-2013 period.[7]

In Tables 1-4, we report the results from regressing those asset-pricing moments on our measure of the R&D content of international trade. The analysis is performed using the more aggregated measure of the R&D content of trade (Tables 1 and 2) as well as the industry measure based on the the input–output matrix (Tables 3 and 4). In either case, we find that the coefficients have signs that are consistent with our previous analysis. More specifically, a 10% increase in the log R&D content of international trade increases the correlation of asset returns by 0.35% (first column of Tables 1) and decreases the volatility of the exchange rate by 0.024% (first column of Table 2) when we use our aggregate measure. With the industry data measure, a 10% increase in the log R&D content of international trade increases the correlation of asset returns by 0.38% (first column of Table 3) and decreases the volatility of the exchange rate by 0.01% (first column of Table 4). Although the coefficients in the Table 4 become non-significant owing to the lack of sufficient variation in our industry-level data, their signs are still negative.[8]

[7]We also capture the idea that R&D propagates at medium-term business cycles, as in Comin and Gertler (2006), by performing the analysis for different frequencies of the asset-pricing moments. We obtain the medium-frequency component of asset prices by applying a bandpass filter that removes frequencies between 100 and 200 quarters. The results, which are available from the authors upon request, are consistent with our main analysis.

[8]These results are robust to the inclusion of time-fixed effects (second column of Tables 1-4).

Table 1: R&D content of international trade and stock market returns

	(1)	(2)
$\log(X_{ij}^{\text{R\&D}})$	0.03562***	0.03300***
	(0.002)	(0.002)
Time fixed effects	No	Yes
Observations	3,420	3,420
R^2	0.090	0.318

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: R&D content of international trade and exchange rate volatility

	(1)	(2)
$\log(X_{ij}^{\text{R\&D}})$	-0.00244***	-0.00282***
	(0.001)	(0.000)
Time fixed effects	No	Yes
Observations	990	990
R^2	0.023	0.252

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: R&D content of international trade(industry) and stock market returns

	(1)	(2)
log($X_{ij}^{R\&D}$)	0.03808***	0.03416***
	(0.008)	(0.006)
Time fixed effects	No	Yes
Observations	306	306
R^2	0.064	0.457

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: R&D content of international trade(industry) and exchange rate volatility

	(1)	(2)
log($X_{ij}^{R\&D}$)	-0.00068	-0.00049
	(0.001)	(0.001)
Time Fixed Effects	No	Yes
Observations	90	90
R^2	0.002	0.216

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2.3 The Margins of International Trade

In this section, we further explore the role of trade in diffusing technology across countries. Recall that we assume technology is embodied in products, and hence diffuses across countries though trade. One can therefore decompose the total value of trade into the number of different products traded across each country-pair—the so called extensive margin of trade—and the amount of each good traded across countries–the so called intensive margin of trade. Our conjecture is that if R&D is transmitted across countries through trade in the products that embody such R&D, then the extensive margin should have more effect (than the intensive margin), on asset-pricing moments.

In Tables 5 and 6, we report the results of regressing those asset-pricing moments on both the extensive margin (EM) and intensive margin (IM) of each country pair's respective international trade. We find that both coefficients are statistically significant, yet the extensive margin accounts for most of the variation in asset-pricing moments. More specifically, if we hold the intensive margin constant, then a 10% increase in the log extensive margin increases the correlation of asset returns by 1.2% (first column of Table 5) and decreases the volatility of the exchange rate by 0.4% (first column column of Table 6); if instead we hold the extensive margin constant, then a 10% increase in the log intensive margin increases the correlation of stock market returns by 0.16% and increases the volatility of the exchange rate by 0.1%. Thus the effect of the intensive margin of trade is weaker than that of the extensive margin. These findings are consistent with the mechanism that we propose.

Table 5: The margins of international trade and stock market returns

	[1]	[2]
log(EM)	0.12636***	0.14925***
	(0.015)	(0.013)
log(IM)	0.01621**	0.00458
	(0.005)	(0.005)
Time fixed effects	No	Yes
Observations	3420	3420
R^2	0.101	0.343

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 6: The margins of international trade and exchange rate volatility

	[1]	[2]
log(EM)	-0.04453***	-0.04106***
	(0.004)	(0.003)
log(IM)	0.00940***	0.00811***
	(0.001)	(0.001)
Time fixed effects	No	Yes
Observations	990	990
R^2	0.129	0.335

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

These results constitute suggestive evidence that innovation–rather than alternative factors driving international trade–plays a unique role in driving the international comovement of asset prices. Indeed, to the extent that products embody technology, variations in the extensive margin of trade reflect variations driven by technological innovation; at the same time, variations in the intensive margin tend to reflect international trade variations driven by factors other than technological innovation. Our findings are consistent with those reported by Liao and Santacreu (2015) concerning the comovement of output growth across countries.

2.4 Robustness

In this section, we perform two robustness exercises. First, we show that our main results about the relationship between asset-pricing moments and R&D embodied in trade are robust to the inclusion of exporter and importer fixed effects. To do that, instead of continuing with our analysis of unique country pair analysis, we compute a measure of R&D trade intensity between importer i and exporter j as

$$X_{ij}^{\text{R\&D}} = R\&D_j \frac{T_{ij}}{GDP_j}$$

Note that, in this case, $X_{i,j}^{\text{R\&D}} \neq X_{j,i}^{\text{R\&D}}$.

We then regress our moments of interest on this measure of R&D embodied in trade, together with importer and exporter fixed effects. The results are consistent with those obtained with unique country pairs and without importer or exporter fixed effects. As Tables 7-10 show, country pairs that share more R&D though international trade have more correlated stock market returns and less volatile exchange rates (regardless of whether we use aggregated data or more disaggregated industry data).

Table 7: R&D content of international trade and stock market returns

	[1]	[2]	[3]	[4]
log($X_{ij}^{\text{R\&D}}$)	0.02884***	0.02605***	0.02196***	0.00952***
	(0.001)	(0.001)	(0.002)	(0.002)
Time fixed effects	No	Yes	No	Yes
Exporter fixed effects	No	No	Yes	Yes
Importer fixed effects	No	No	Yes	Yes
Observations	6,818	6,818	6,818	6,818
R^2	0.073	0.304	0.225	0.462

*Note:*Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: R&D content of international trade and exchange rate volatility

	[1]	[2]	[3]	[4]
log($X_{ij}^{\text{R\&D}}$)	-0.00209***	-0.00243***	-0.00638***	-0.00817***
	(0.000)	(0.000)	(0.001)	(0.001)
Time fixed effects	No	Yes	No	Yes
Exporter fixed effects	No	No	Yes	Yes
Importer fixed effects	No	No	Yes	Yes
Observations	1,970	1,970	1,970	1,970
R^2	0.019	0.248	0.276	0.521

*Note:*Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 9: R&D content of international trade (industry) and stock market returns

	[1]	[2]	[3]	[4]
log($X_{ij}^{\text{R\&D}}$)	0.03429***	0.03076***	0.06798***	0.03477**
	(0.006)	(0.004)	(0.016)	(0.012)
Time fixed effects	No	Yes	No	Yes
Exporter fixed effects	No	No	Yes	Yes
Importer fixed effects	No	No	Yes	Yes
Observations	612	612	612	612
R^2	0.058	0.452	0.188	0.576

*Note:*Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: R&D content of international trade (industry) and exchange rate volatility

	[1]	[2]	[3]	[4]
log($X_{ij}^{\text{R\&D}}$)	-0.00062	-0.00044	-0.00984***	-0.00865***
	(0.001)	(0.001)	(0.002)	(0.002)
Time fixed Effects	No	Yes	No	Yes
Exporter fixed effects	No	No	Yes	Yes
Importer fixed effects	No	No	Yes	Yes
Observations	180	180	180	180
R^2	0.002	0.215	0.323	0.520

*Note:*Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, we take our measure of the R&D content of international trade ($X_{ij}^{\text{R\&D}}$) and the component that measures the total value international trade–$\frac{TI_{ij} + TI_{ji}}{GDP_i+GDP_j}$–and regress our moments of asset-pricing moments on these two components. The goal is to investigate whether, including the total value of international trade across countries, removes the significance of the R&D content of international trade. We report the regression results in Table 11, which shows that this is not the case. In fact, we find that R&D remains a significant driver of asset price comovement even when the regression incorporates the total value of international trade.

Table 11: R&D content of international trade, total value of international trade and stock market returns

	Stock market correlation	Exchange rate volatility
log($X_{ij}^{\text{R\&D}}$)	0.00881***	-0.00319***
	(0.003)	(0.001)
$\log(\frac{TI_{ij}+TI_{ji}}{GDP_i+GDP_j})$	0.01456***	-0.00556***
	(0.003)	(0.001)
Time fixed effects	Yes	Yes
Exporter fixed effects	Yes	Yes
Importer fixed effects	Yes	Yes
Observations	5,301	1,530
R^2	0.703	0.693

*Note:*Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

3 Model

In this section, we present a model of innovation and international diffusion of R&D via trade in varieties that captures our empirical findings and accounts for a range of international macroeconomics and asset-pricing moments. Each country has a representative household, with recursive preferences, that consumes a final good. A final good producer uses labor, capital, and a composite of intermediate goods—which we call materials—to produce a non-tradable final good that is used for consumption, investment in capital, and investment in R&D. Materials are produced using traded intermediate goods (varieties), both domestic and foreign, which are produced by monopolistic competitive firms. The production of materials reflects a love-of-variety effect; thus if expenditures are held constant then a higher number of varieties increases the country's aggregate productivity. New varieties are introduced in each country through an endogenous process of innovation, after which they spread exogenously across countries through a slow process of adoption. Endogenous innovation and exogenous adoption, along with recursive preferences, are the new features at the core of our mechanism.

Financial markets are assumed to be internationally complete.

Next, we describe the domestic economy d; the foreign economy f is defined analogously. Throughout the paper, a variable's subscript refers to the origin country — the exporter — while its superscript refers to the destination country — the importer.

3.1 Households

The domestic representative household has Epstein and Zin (1989) recursive preferences over consumption:

$$U_{d,t} = \{(1-\beta)C_{d,t}^{\theta} + \beta(E_t[U_{d,t+1}^{1-\gamma}])^{\theta/(1-\gamma)}\}^{1/\theta}; \tag{2}$$

In this expression: U is utility, t denotes time, β is the subjective discount factor, C denotes consumption, E is the expectations operator, γ is the constant relative risk aversion (CRRA), $\theta = \frac{1-\gamma}{1-1/\psi}$, and $\psi \equiv \frac{1}{1-\theta}$ is the intertemporal elasticity of substitution (IES). We assume that $\psi > 1/\gamma$, so that the representative agent has a preference for early resolution of uncertainty and fears variations in the economy's long-run prospects.

The stochastic discount factor is given by

$$M_{d,t+1} = \beta \left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1} \left(\frac{U_{d,t+1}}{E_t[U_{d,t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1-\gamma-\theta}, \tag{3}$$

Here the right hand side's first term, $\beta\left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1}$, captures short-run risk. We refer to this term in logs as m_d^{SR}. The last term, $\left(\frac{U_{d,t+1}}{E_t[U_{d,t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1-\gamma-\theta}$, captures the agent's concerns about the uncertainty in future growth. In logs, we refer to it as m_d^{LR}. The household consumes, supplies labor to the final good producer, and makes investment and savings decisions while participating in complete international financial markets. The household budget constraint is accordingly

$$C_{d,t} + E_t\Big[M_{d,t+1}\mathcal{A}_{d,t+1}\Big] = W_{d,t}L_{d,t} + \mathcal{A}_{d,t},$$

where $W_{d,t}$ is the wage rate, $L_{d,t}$ denotes hours worked, and $\mathcal{A}_{d,t}$ is the state-contingent value of the household's financial wealth. There is no disutility of labor, and so the household supplies its entire endowment, which is normalized to one.

3.2 Final Good Producer

Domestic final producers are perfectly competitive. They use capital ($K_{d,t}$), labor ($L_{d,t}$), and a composite of domestic and foreign intermediate goods ($G_{d,t}$), to produce a nontraded final good ($Y_{d,t}$) in accordance with the following Cobb–Douglas production function:

$$Y_{d,t} = \left(K_{d,t}^{\alpha} \left(\Omega_{d,t} L_{d,t}\right)^{(1-\alpha)}\right)^{(1-\xi)} G_{d,t}^{\xi}. \tag{4}$$

The composite good $G_{d,t}$ is defined as

$$G_{d,t} = \left[h \left(\sum_{i=1}^{N_{d,t}^{d}} (X_{d,i,t}^{d})^{\nu}\right) + (1-h) \left(\sum_{i=1}^{N_{f,t}^{d}} (X_{f,i,t}^{d})^{\nu}\right)\right]^{1/\nu}; \tag{5}$$

Here $X_{d,i,t}^{d}$ is the amount of domestically produced intermediate good i that is used for final production in the domestic economy, $X_{f,i,t}^{d}$ is the amount of foreign-produced intermediate good i that is used for final production in the domestic economy, $N_{d,t}^{d}$ (resp. $N_{f,t}^{d}$) is the mass of domestic (resp. foreign) intermediate goods that is used by domestic final producers, and $1/(1-\nu)$ is the elasticity of substitution across intermediate goods (with $\nu < 1$). The parameter h captures the degree of home bias. The parameter α represents the physical capital share, and ξ is the share of materials in final production. Intermediate goods are aggregated according to a constant elasticity of substitution' production function (à la Ethier).

The exogenous process $\Omega_{d,t}$ is the only source of exogenous uncertainty in our model. We assume that $\Omega_{d,t} = \exp a_{d,t}$, where $a_{d,t}$ follows the first-order autoregressive, or AR(1), process

$$a_{d,t} = \varphi \; a_{d,t-1} + \varepsilon_{d,t},$$

where $\varepsilon_{d,t} \sim N(0, \sigma^2)$. We allow for cross-country correlation in the exogenous technology shocks and let $\rho = \mathrm{corr}(\varepsilon_{d,t}, \varepsilon_{f,t})$.[9]

Final producers choose capital, labor, investment, and intermediate goods to maximize

[9] When solving the model, we augment the AR(1) process for exogenous technology with an error correction term in the spirit of Colacito and Croce (2013). This term ensures the stability of our solution method yet has virtually no effect on the results.

shareholder value subject to the production technology (4). Formally, we have

$$\max_{\{I_{d,t},L_{d,t},K_{d,t+1},X^d_{d,i,t},X^d_{f,i,t}\}_{t\geq 0,}} E_0\left[\sum_{t=0}^{\infty} M_{d,t}D_{d,t}\right], \tag{6}$$

where the firm's dividends are given by

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - \sum_{i=1}^{N^d_{d,t}} P^d_{d,i,t}X^d_{d,i,t} - \sum_{i=1}^{N^d_{f,t}} P^d_{f,i,t}X^d_{f,i,t}. \tag{7}$$

Here $M_{d,t}$ is the stochastic discount factor, $W_{d,t}$ is the wage rate, $I_{d,t}$ is investment in physical capital, $P^d_{d,i,t}$ is the price of a domestically produced intermediate good, and $P^d_{f,i,t}$ is the price of a foreign-produced intermediate good that is used for domestic production. Both prices are expressed in units of the domestic producer's final good.

The law of motion for physical capital is given by

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right)K_{d,t}, \tag{8}$$

where $\delta \in (0,1)$ is the depreciation rate and where $\Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right)$ captures convex capital adjustment costs.[10]

3.3 Intermediate Good Producers

In each country, a set of monopolistic competitive firms produces a differentiated good using final output according to a constant returns to scale (CRS) production function, that is, one unit of the final output is used to produce one unit of the intermediate good. All intermediate producers produce with the same efficiency. Intermediate producers produce for both the domestic and foreign market. Every period, each domestic intermediate producer i solves

[10] As in Jermann (1998), $\Lambda_{d,t} \equiv (I_{d,t}/K_{d,t}) = (\alpha_1/\zeta)(I_{d,t}/K_{d,t})^{\zeta} + \alpha_2$. The parameters α_1 and α_2 are chosen so that there are no adjustments costs in the steady state, and $1/(1-1/\zeta)$ is the elasticity of the investment rate with respect to Tobin's Q.

the following static profit maximization problem:

$$\begin{aligned}
\max_{P^d_{d,i,t},P^{f*}_{d,i,t}} \Pi_{d,i,t} &\equiv \max_{P^d_{d,i,t},P^{f*}_{d,i,t}} (\pi^d_{d,i,t} + \pi^f_{d,i,t}) \\
&= \max_{P^d_{d,i,t}} \quad P^d_{d,i,t} \quad X^d_{d,i,t}(P^d_{d,i,t}) - X^d_{d,i,t} \quad (P^d_{d,i,t}) \\
&+ \max_{P^{f*}_{d,i,t}} \underbrace{(P^f_{d,i,t}Q_t)}_{P^{f*}_{d,i,t}} \; X^f_{d,i,t}(\underbrace{P^f_{d,i,t}Q_t}_{P^{f*}_{d,i,t}}) - X^f_{d,i,t}(\underbrace{P^f_{d,i,t}Q_t}_{P^{f*}_{d,i,t}}),
\end{aligned} \tag{9}$$

where $\pi^d_{d,i,t}$ (resp. $\pi^f_{d,i,t}$) are the profits from selling the domestic product at home (resp. abroad) and where $P^{f^*}_{d,i,t} = P^f_{d,i,t}Q_t$ is the price (in units of the domestic good) of a domestically produced intermediate good that is being exported. We use Q_t to denote the real exchange rate, defined as the number of domestic final goods per unit of foreign final good.[11]

3.4 Innovation and Adoption

3.4.1 Innovation

In each country, innovators invest resources (final output) to introduce new prototypes of a product. A successful innovator starts producing the new good as an intermediate producer. Each domestic innovator i chooses $S_{d,i,t}$ units of final output to maximize the present discounted value of future profits that it expects to obtain from selling the good to both domestic and foreign producers.

The law of motion for new prototypes is

$$N^d_{d,t+1} = \vartheta_{d,t}S_{d,t} + (1-\phi)N^d_{d,t}; \tag{10}$$

where $N^d_{d,t}$ is the mass of new technologies arriving in country d at time t, ϕ is the exogenous probability that a new variety becomes obsolete, and $\vartheta_{d,t}$ is the productivity of innovation. Following Comin and Gertler (2006), we assume that this productivity takes the following functional form:

[11] We express real prices of the intermediate goods in units of the importer's final good. This means that, when the domestic (resp. foreign) intermediate good is used for the production of the foreign (resp. domestic) final output, $P^f_{d,t} \equiv (1/\nu)Q_t^{-1}$ and $P^d_{f,t} \equiv (1/\nu)Q_t$.

$$\vartheta_{d,t} = \frac{\chi(N^d_{d,t} + N^d_{f,t})}{S^{1-\eta}_{d,t}(N^d_{d,t})^\eta}. \tag{11}$$

The process $S_{d,t} = \sum_{i=1}^{N^d_{d,t}} S_{d,i,t}$ represents total R&D expenditures in the domestic country (in terms of the domestic final good). The parameter η denotes the elasticity of innovation with respect to R&D, and χ is a scaling parameter. The term $N^d_{d,t} + N^d_{f,t}$ captures a spillover effect on the innovation of country d. The spillover stems from two sources: (i) the varieties that have already been invented by country d; and (ii) the varieties invented in country f that are now used by country d. In other words, an innovator can learn from its own innovations, and also from foreign innovations that arrive through imports. In this specification, $\vartheta_{d,t}$ is an externality; it is taken as given when innovators choose their optimal investment in R&D.

3.4.2 International Adoption

In every period, only a fraction ϑ^f_d of intermediate goods from country d can be used by the final good producer in country f. The parameter ϑ^f_d, which governs the speed of adoption, is crucial in our mechanism. The law of motion for domestic intermediate goods that can be used by the foreign final producer evolves according to

$$N^f_{d,t+1} - (1-\phi)N^f_{d,t} = \vartheta^f_d(1-\phi)(N^d_{d,t} - N^f_{d,t}), \tag{12}$$

where $N^f_{d,t}$ is the number of domestic goods imported by the foreign economy. If follows that the mass of domestic varieties not yet adopted by the foreign country is equal to $N^d_{d,t} - N^f_{d,t}$.

3.4.3 Value Functions

The value of a domestic innovation, $V_{d,i,t}$, is given by the present discounted value of the profits that innovator i expects to obtain from selling the good domestically and abroad. Let the value of the domestic innovations that are immediately sold to domestic (resp. foreign) final producers be $V^d_{d,i,t}$ (resp. $V^f_{d,i,t}$), and let the value of the innovations that could be adopted by country f be $J^f_{d,i,t}$. Then

$$V_{d,i,t} = V^d_{d,i,t} + J^f_{d,i,t}, \tag{13}$$

where

$$\begin{aligned} V^d_{d,i,t} &= \pi^d_{d,i,t} + (1-\phi)E_t[M_{d,t+1}V^d_{d,i,t+1}], &(14)\\ V^f_{d,i,t} &= \pi^f_{d,i,t} + (1-\phi)E_t[M_{d,t+1}V^f_{d,i,t+1}], &(15)\\ J^f_{d,i,t} &= (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta^f_d V^f_{d,i,t+1} + (1-\vartheta^f_d)J^f_{d,i,t+1}\right)\right]. &(16) \end{aligned}$$

According to the value function (16), with probability ϑ^f_d the firm will sell the product abroad at $t+1$, while with probability $1-\vartheta^f_d$ the product will not be adopted.

Discounted future profits on patents are the payoff to innovators. Because the R&D sector is competitive, the free-entry condition for R&D investment–in the symmetric equilibrium under which all firms are identical is

$$S_{d,t} = E_t\left[M_{d,t+1}V_{d,t+1}\right]\left(N^d_{d,t+1} - (1-\phi)N^d_{d,t}\right) \tag{17}$$

or, equivalently,

$$\frac{1}{\vartheta_{d,t}} = E_t\left[M_{d,t+1}V_{d,t+1}\right]. \tag{18}$$

3.5 Resource Constraint

Final output is used for consumption, intermediate goods production, and investment in R&D. Hence we may write the resource constraint as

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N^d_{d,t}X^d_{d,t} + N^f_{d,t}X^f_{d,t}\ .$$

3.6 Equilibrium and Steady State

We define a symmetric equilibrium as a set of equations according to which all firms within a country behave symmetrically.

For each country $i = (d, f)$, a *general symmetric equilibrium* is defined as an exogenous stochastic sequence of technology shocks $\{\Omega_{i,t}\}^{\infty}_{t=0}$, an initial vector $\{N^d_{d,0}, N^f_{d,0}, N^f_{f,0}, N^d_{f,0},$ $K_{d,0}, K_{f,0}\}$, a set of parameters $\{\beta, \theta, \gamma, \psi, \alpha, \xi, \varphi, \sigma, \rho, \delta, \theta, \nu, \chi, \phi, \eta, h, \vartheta^f_d\}$, a sequence of aggregate prices $\{W_{i,t}, V_{i,t}, Q_t, q_{it}\}^{\infty}_{t=0}$, value functions $\{V^i_{it}, J^f_{dt}, J^d_{ft}, V^f_{dt}, V^d_{ft}\}^{\infty}_{t=0}$, a sequence of intermediate good prices $\{P^d_{d,t}, P^f_{d,t}, P^f_{f,t}, P^d_{f,t}\}^{\infty}_{t=0}$, a sequence of aggregate quantities $\{Y_{i,t}, G_{i,t}, C_{i,t},$ $I_{i,t}, L_{i,t}, S_{i,t}\}^{\infty}_{t=0}$, quantities of intermediate goods $\{X^d_{d,t}, X^f_{d,t}, X^f_{f,t}, X^d_{f,t}\}^{\infty}_{t=0}$, a sequence of profits $\{\Pi_{d,t}, \Pi_{f,t}, \pi^d_{d,t}, \pi^f_{d,t}, \pi^f_{f,t}, \pi^d_{f,t}\}_{t=0}^{\infty}$, and laws of motion $\{N^d_{d,t+1}, N^f_{d,t+1}, N^f_{f,t+1}, N^d_{f,t+1}, K_{i,t+1}\}_{t=0}^{\infty}$ such that the following conditions hold:

- The state variables satisfy their respective laws of motion.
- The endogenous variables solve the producers', innovators', and representative households' problems.
- The resource constraint is satisfied.
- Prices are such that all markets clear.

The equilibrium conditions are given in Appendix B.

3.7 Aggregate Productivity

Domestic aggregate productivity can be expressed as

$$Z_{d,t} \equiv \Omega_{d,t} \left(\tilde{A}\right)^{\frac{1}{1-\alpha}} \left[hN^d_{d,t} + \left(\frac{h}{1-h} Q_t\right)^{\frac{\nu}{\nu-1}} N^d_{f,t} \right], \tag{19}$$

where $\tilde{A} \equiv (\xi\nu)^{\xi/(1-\xi)}$. Taking logarithms, we have

$$\log Z_{d,t} = \log \Omega_{d,t} + \log \left\{ \left(\tilde{A}\right)^{1/(1-\alpha)} \left[hN^d_{d,t} + \left(\frac{h}{1-h} Q_t\right)^{\frac{\nu}{\nu-1}} N^d_{f,t} \right] \right\}.$$

Thus aggregate productivity has both an exogenous and an endogenous component, that is

$$\log(Z_{d,t}) = \log(\mathrm{Z}^{\mathrm{EXO}}_{d,t}) + \log(\mathrm{Z}^{\mathrm{ENDO}}_{d,t}).$$

Here

$$\log(\mathrm{Z}^{\mathrm{EXO}}_{d,t}) \equiv \log \Omega_{d,t}$$

and

$$\log(\mathrm{Z}^{\mathrm{ENDO}}_{d,t}) \equiv \log \left\{ \left(\tilde{A}\right)^{1/(1-\alpha)} \left[hN^d_{d,t} + \left(\frac{h}{1-h} Q_t\right)^{\frac{\nu}{\nu-1}} N^d_{f,t} \right] \right\} \quad .$$

The exogenous component of aggregate productivity is given by the stochastic process $\Omega_{d,t}$; the endogenous component, which is the driver of growth in our model, depends on the number $N_{d,t}^d$ of varieties produced domestically and the number $N_{f,t}^d$ of varieties produced in the foreign country and that have been already adopted in the home country.

Inspecting equation (19) reveals that expected future productivity is driven by fluctuations in the number of varieties, both domestic and foreign. These fluctuations, in turn, reflect both the value of an innovation, $V_{d,t} = V_{d,t}^d + J_{d,t}^d$, and R&D expenditures, $S_{d,t}$ (*cf.* equations 10 and 17). These expenditures are persistent in the data and, as we show in Section 4, strongly affect asset prices and their comovement across countries.

3.8 Dividends and the Stock Market

Stocks are viewed as claims on all the production sectors: the final good sector, the intermediate good sector, and the innovation sector. Hence the aggregate dividend is defined as the net payout from these production sectors; formally,

$$\mathcal{D}_{d,t} = D_{d,t} + N_{d,t}^d \pi_{d,t}^d + N_{f,t}^d \pi_{f,t}^d - S_{d,t}. \tag{20}$$

Optimality implies the following asset-pricing condition:

$$\mathcal{P}_{d,t} = E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})],$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price and $\mathcal{D}_{d,t}$ is the aggregate market dividend. In Appendix C, we show how to derive the price of the stock market as the present discounted value of all future dividends, following Comin, Gertler, and Santacreu (2009) who derive a similar expression for a closed economy.

3.9 Exchange Rate, and Risk Sharing

Since we assume that financial markets are complete, the exchange rate depreciation is given by the ratio of foreign to domestic stochastic discount factors; thus,

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{f,t+1}}{M_{d,t+1}}. \tag{21}$$

Because preferences are recursive, the risk-sharing mechanism is nonstandard. In particular, agents fear not only current shocks but also variation in future utility. Formally, let $\Upsilon_t =$

$Q_t \left(\frac{C_{d,t}}{C_{f,t}}\right)^{\theta-1}$. Using the stochastic discount factor from equation (3) together with the no-arbitrage condition (21), we can express Υ_t recursively as follows:

$$\Upsilon_{t+1} = \Upsilon_t \, \frac{M_{f,t+1}}{M_{d,t+1}} \, \frac{\exp{(\theta-1)\Delta c_{d,t+1}}}{\exp{(\theta-1)\Delta c_{f,t+1}}} \quad . \tag{22}$$

Observe that Υ_t is constant in the CRRA case. Under Epstein–Zin recursive preferences, however, Υ_t evolves as a function of the cross-country realizations of agents' continuation utilities. Colacito and Croce (2013) provide a thorough analysis of this mechanism.

4 The Mechanism

We illustrate the model's main mechanisms by analyzing the effect of an increase in domestic aggregate productivity on asset prices. After an increase in such productivity, final producers demand more intermediate goods–both domestic and foreign. The resulting higher demand for intermediate goods increases the value of domestic innovation, and this leads to more resources being allocated to domestic R&D.

When preferences are recursive, risky growth through endogenous innovation determines the optimal level of R&D and thus also the level of current and future expected growth. Innovations then spread across countries via a process of technology adoption that we measure in terms of trade in varieties. As shown in equation (17), optimal R&D spending equals the present discount value of an innovation. With recursive preferences the discount factor is volatile and highly correlated across countries. Growth is risky and a significant share of this risk is global. The result is a first order effect on the international correlation structure of stock market returns and on the volatility of exchange rates.

While recursive preferences have a large effect on asset prices, their impact on realized consumption growth is limited. This is the combined result of two channels. The first is the well-known result that, in a standard real business cycle model, decoupling risk aversion from the elasticity of intertemporal substitution significantly improves the model's ability to replicate asset-pricing moments while leaving quantity moments virtually unaltered (see Tallarini (2000) and Kaltenbrunner and Lochstoer (2010)). Unlike those papers, our model features endogenous growth, so that we observe *some* effects on quantities, as highlighted by Kung and Schmid (2015).

The second channel comes from the international risk sharing mechanism with complete markets. After a positive domestic productivity shock, the domestic economy becomes more innovative, hence expected future productivity increases. In turn, the long-run growth prospects of the economy improve. With recursive preferences, agents experience a substantial drop in their marginal utility because they fear variations in future utility that are associated with news about future growth. Complete financial markets then imply that domestic agents are willing to shift a substantial amount of resources to the foreign economy. The outcome is a cross-country correlation in consumption that is lower than what we would obtain in the absence of our recursive risk-sharing mechanism. This is the mechanism emphasized by Colacito and Croce (2013) for the case of an endowment economy. In contrast, stock market returns become even more highly correlated, since they reflect the present discounted value of future growth prospects. These prospects are shared across countries, as captured by the high cross-country correlation of the long-run components of the stochastic discount factors.

One must bear in mind that, in our model, the wedge in the dynamics between consumption and asset prices is driven mainly by the effect of expected future productivity growth. Indeed, and consistent with previous studies (e.g., Kehoe and Ruhl 2013), quantities are little affected by increases in realized current aggregate productivity.

4.1 Impulse Response Analysis

We study impulse response functions in order to inspect more closely our model's main mechanism and also to emphasize the role of recursive preferences. Figures 3 and 4 plot the reaction of the main endogenous variables following a one-standard deviation positive productivity shock. To streamline our analysis, our shocks are orthogonalized.

After a positive productivity shock at home, domestic R&D increases, because then higher demand of domestic final producers increases the value of domestic innovations. This increase is greater under recursive preferences than under CRRA, since risky growth affects the optimal level of investment in R&D (see equation (17)). The effect on the foreign economy depends on the type of preferences considered. Upon impact, in response to such productivity shock, foreign R&D spending falls below its steady-state level regardless of the preference structure. The consequent higher demand of domestic final producers for foreign intermediate goods induces a reallocation of resources away from innovation and towards the production of varieties to be sold abroad. But when preferences are recursive, this pattern is reversed after a few periods owing to an increase in the prospects of future expected productivity growth.

In our model, the economy's growth rate can be expressed as the present discounted value of domestic and foreign profits:

$$\Delta N^{d}_{d,t+1} = (1-\phi) + E_t \left[\chi^{\frac{1}{\eta}} \sum_{j=1}^{\infty} M_{d,t+j} (1-\phi)^{j-1} (\pi^{d}_{d,t+j} + \pi^{f}_{d,t+j}) \right]^{\frac{\eta}{1-\eta}},$$

where $M_{d,t+j} \equiv \prod_s^j M_{d,t+s}$ is the j-period stochastic discount factor.[12] The long-run component of the stochastic discount factor is highly correlated across countries and captures the idea that future growth prospects are positive not only in the domestic economy but also in the foreign economy. Eventually, this results in an increase in foreign R&D spending above its steady-state. Under CRRA, long-run risk is not priced; hence, the latter effect is muted and R&D spending remains below its steady-state until it converges back.

[12] Kung and Schmid (2015) derive a similar expression in the context of a closed economy.

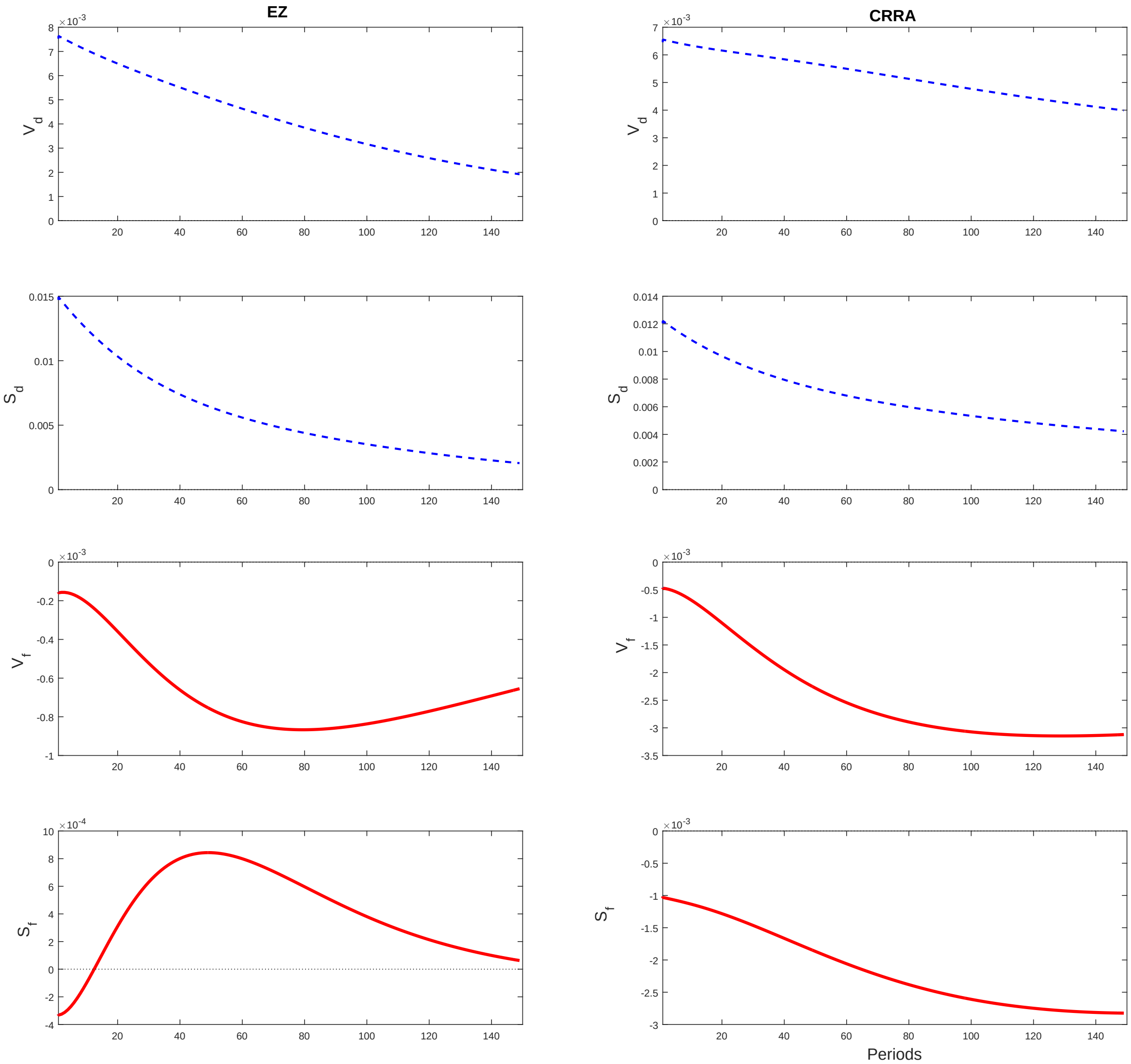

Figure 3: This figure plots impulse response functions in the baseline model with recursive preferences (left panel) and under CRRA (right panel) for R&D spending, s, and the value of an innovation, v. Lower case letters denote logs.

These fluctuations in the value of an innovation and in R&D spending have a very different effect on consumption and asset prices. Once again, the type of preferences we consider plays a crucial role. Indeed, with recursive preferences (EZ), domestic consumption increases following higher productivity at home, whereas foreign consumption slightly decreases. This outcome is a consequence of the following mechanism. First, endogenous innovation generates substantial volatility in the economy's growth prospects, transforming the short-run productivity shock into a positive news shock with a long-run impact on expected future productivity; thus, expected future productivity and expected future consumption are both high. Second, the positive news shock becomes *global* because the foreign country expects there will be an increased number of technologies imported from the domestic country. Third, the foreign agent's marginal utility declines following positive news about future productivity growth. Fourth, optimal risk sharing implies a slight reduction in foreign consumption, as the foreign agent diverts resources (final output) into the production of intermediate goods to be sold to the domestic final producers.

In the CRRA case, the risk sharing mechanism is standard despite the presence of endogenous innovation. Risks associated with variations in the economy's future prospects are not priced and hence have no effect on the foreign agent's marginal utility. Risk sharing dictates that some of the resources available in the domestic economy be shipped abroad for foreign consumption. Note that, in the foreign economy, the reallocation of resources to the production of intermediate goods is less substantial under CRRA than under EZ preferences. This result is due to the weaker effect that a domestic productivity shock has on the future growth prospects of the foreign economy.

The positive correlation among the stochastic discount factors generated by the endogenous mechanism of our baseline model with recursive preferences affects the dynamics of exchange rates and stock market returns. In particular, exchange rates are less volatile than in the standard CRRA case. It is noteworthy that, as we will show in our main calibration in Section 5, this reduced volatility is not obtained at the cost of counterfactually high cross-country correlation in consumption growth or of low Sharpe Ratios. Finally, stock market returns reflect the evolution of stock prices as described in Section 3.8. As good news about future productivity spreads across countries, we observe an increase in the comovement of their economies' future growth prospects that leads to increased cross-country correlation of stock market returns.

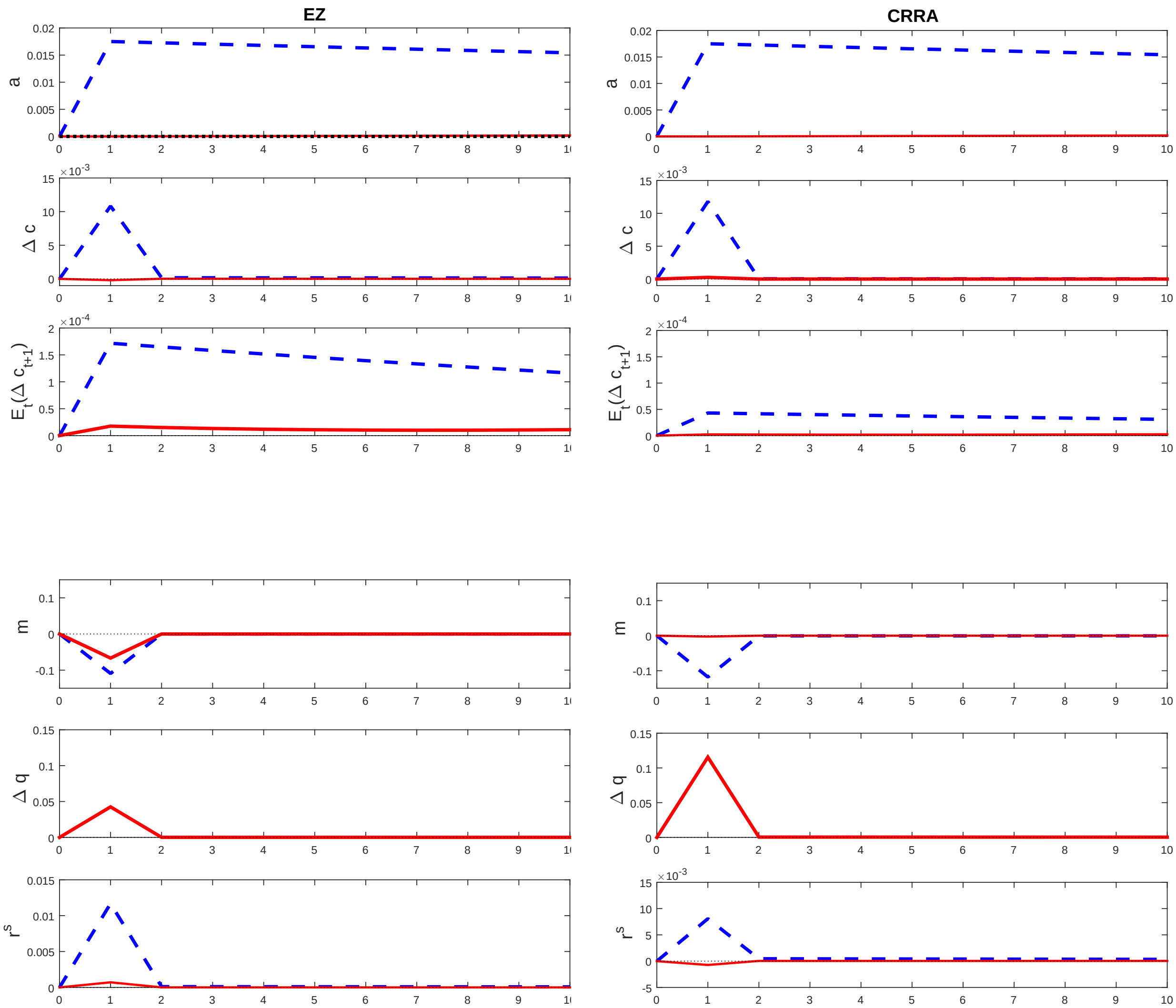

Figure 4: This figure plots impulse response functions in the baseline model under recursive preferences (left panel) and under CRRA (right panel)–for the exogenous component of technology a, the consumption growth Δc, the expected consumption growth $E_t \Delta c_{t+1}$, the stochastic discount factor m, the depreciation rate Δq, and the return on the stock market r^S to a one-standard deviation positive productivity shock. The shock is orthogonalized. Lower case letters denote logs.

5 Quantitative Implications

In this section we discuss the quantitative implications of our model and explore its ability to replicate key international moments for macroeconomic quantities, stock market returns, and exchange rates. Our baseline model is calibrated at a quarterly frequency.

5.1 Calibration

We need to specify a total of 16 parameters, whose values are reported in Table 12. We start by discussing those that are relatively more standard. Values for the preference parameters are set in the spirit of the long-run risk literature (e.g., Bansal and Yaron 2004; Colacito and Croce 2013). In particular, we set the coefficient γ of relative risk aversion equal to 10 and the coefficient θ equal to 0.4, which implies an IES value of $\psi = 1.66$. Under this calibration of the preference parameters, agents in the economy fear shocks to expected future growth. The subjective discount factor is chosen so as to fix the mean of the risk-free rate; thus, $\beta = 0.984^{1/4}$.

The parameters relating to the technology for final good production are obtained from Comin and Gertler (2006) and Kung and Schmid (2015). The capital share α is set to 0.35 to match the average capital share, and the share ξ of intangible capital is set to 0.5. The depreciation rate of physical capital is set to 0.02 and the parameter ζ is chosen to fix an elasticity of the investment rate with respect to Tobin's Q of 0.3. We follow the literature and use a value of 0.7 for ν; this parameter is related to the elasticity of substitution across intermediate goods and fixes the intermediate good markup in our model. The home bias parameter, h we set to 0.97, in line with the international estimates of Colacito and Croce (2013).

We set the autocorrelation of the exogenous technology shock to $0.95^{1/4}$ and the volatility parameter σ at 1.75% in order to obtain a realistic volatility for consumption and output growth. Finally, we fit an AR(1) process to each of our sample country's total factor productivity and compute the cross-country correlation of the error term; the resulting correlation (in the exogenous shocks) is 0.35.

We now discuss the less standard parameters that govern the process of innovation and technology adoption. The parameter χ is a pure scaling parameter; we set its value such that the steady-state growth rate of consumption has an annualized mean of 1.90%. Together

Table 12: Parameters for the baseline quarterly calibration.

Description	Parameter	Value
Preferences		
Risk aversion	γ	10
IES	$\psi = 1/(1-\theta)$	1.66
Subjective discount factor	β^4	0.984
Final Production		
Capital share	α	0.35
Share of materials	ξ	0.5
Autocorrelation of $\Omega = e^a$	φ^4	0.95
Volatility of exogenous shock ε	σ	1.75%
Cross-correlation of exogenous shock	ρ	0.35
Depreciation of capital stock	δ	0.02
Investment adjustment cost	$1/(1-1/\zeta)$	0.3
Inverse markup	ν	0.7
Innovation and International Adoption		
Scale	χ	0.4240
Innovation obsolesce rate	ϕ	0.0375
Elasticity of innovation w.r.t. R&D	η	0.50
Home bias	h	0.97
International adoption	$\vartheta_d^f \times 4$	0.010

with ν's value, χ then gives us a value of $\tilde{A}$ that is consistent with the balanced growth restriction. The parameter η governs the elasticity of new varieties with respect to R&D and is set to 0.5, a number within the range of estimates given by Griliches (1990). Finally, we set $\phi = 0.0375$, which corresponds to a 15% annualized depreciation rate of the R&D stock.

The last parameter we calibrate, ϑ_d^f, governs the speed of international adoption. We use annual data on R&D together with disaggregated bilateral trade data to obtain a value for this parameter. Thus, for each country pair, we compute the change in the extensive margin of trade, which is defined as the number of products traded among each country pair. We regress this variable against the R&D expenditures of that country-pair's exporter. The coefficient derived from that regression captures the effect of an exporter's R&D expenditures on the rate at which its product can be adopted by the importer; this determines the adoption rate ϑ_d^f. Our estimation delivers an annual value of 0.010 for that rate, which is what we use in our baseline calibration. The 95% interval is $[0.008, 0.144]$, and we use the upper bound of this interval to investigate the effect of an increase in the speed of international adoption.

Given these parameters, we use perturbation methods to solve our system of equations. We compute a third-order approximation of our policy functions using the Dynare++ package. All variables included in our code are expressed in log-units.[13]

5.2 Results

Table 13 reports simulated moments of four different calibrations: baseline, CRRA, EXO, and Fast Adoption. For the CRRA calibration, we impose $\psi = 1/\gamma$ and leave all other parameters unaltered. The EXO calibration corresponds to a model in which innovation is exogenous.[14] For the Fast Adoption calibration, we increase ϑ_d^f from 0.010/4 to 0.144/4. Consistently with our empirical analysis, the length of our sample is 72 quarters; all results are averaged over 1,000 simulations.

In terms of the moments of macroeconomic quantities, our baseline model with recursive preferences can generate reasonable means and standard deviations for both output and consumption growth. The mean growth rate is 1.90% for both variables, and their respective standard deviations are 2.13% and 2.53%. Note that our model endogenously generates an extremely high autocorrelation in the conditional mean of future consumption growth and total TFP growth, which suggests the existence of a slow-moving component that governs future growth prospects (cf. Bansal and Yaron 2004). Table 13 also reports moments of the properties of innovation and R&D expenditure. The cross-country correlation of R&D intensity is about 0.32 in our calibration and approximately 0.4 in the data. Similar values are obtained for the cross-country correlation of the growth rate in the number of varieties.

The model generates a cross-country correlation of consumption growth of 0.32. That relatively low correlation is in line with the empirical estimates of Section 2 and also with previous studies. This value is lower than the calibrated cross-country correlation of exogenous productivity shocks (0.35)—a consequence of the risk-sharing mechanism implied by recursive preferences. When a positive short-run shock hits the domestic economy, innovation and endogenous growth generate long-term growth effects that propagate to the rest of the world. So, under recursive preferences, foreign marginal utility drops substantially and

[13]For additional details on the solution and on the approximation of recursive economies with multiple agents see, Colacito and Croce (2012, 2013) and Rabitsch, Stepanchuk, and Tsyrennikov (2015).

[14]In this version of the model, new prototypes arrive exogenously according to a Poisson process and so the steady-state growth rate of consumption does not change; hence growth is exogenous.

results in a reallocation of resources toward the production of foreign intermediate goods, slightly reducing foreign consumption.

Our model performs quite well in terms of the dynamics of R&D expenditure, which are volatile and persistent. The volatility of Δs is 2.96% (3% in the data) and the autocorrelation of S/N is 0.90 (0.75 in the data). The cross country correlation is 0.30, also in line with the data (0.27). Innovation coming from R&D expenditure is the source of growth and its dynamics are crucial for the asset-pricing implications of our model.

Focusing now on the two main asset-pricing moments of our analysis, we see that our model generates a volatility in exchange rates of 9.72% and a cross-country correlation in stock market returns (r^{s}) as high as 0.66. These results are a direct consequence of the joint presence of recursive preferences and a sizable amount of endogenous long-run growth uncertainty, where the latter spreads across countries through our international adoption mechanism. Recursive preferences decouple agents' marginal rates of substitution from realized consumption growth. Endogenous growth and international adoption result in significant comovement in agents' expected future utilities; the result is a less volatile depreciation rate, and greater cross-country correlation in stochastic discount factors (cf. equation 21). Whereas the short-run component of the stochastic discount factor, m^{SR}, is as correlated as consumption growth across countries, the long-run component, m^{LR}–which is generated from endogenous growth and international diffusion of technology–is highly volatile (28%) and strongly correlated across countries (0.956).

We remark that our model generates a low and persistent risk-free rate, as observed in the data. As is common in international production models, its volatility is lower than what we observe in the data. The Sharpe ratio for the stock market, $\mathrm{Std}(m)/E[m]$, is in line with the data and equal to 0.30. While sizeable, the premia and volatilities of returns in the model do not rationalize their empirical counterparts entirely. However, those moments are obtained assuming a relatively low leverage of 3, in the spirit of Boldrin, Christiano, and Fisher (2001). A similar issue arises in Kung and Schmid (2015) and in Colacito, Croce, Ho, and Howard (2012), and can be considerably alleviated by increasing the level of financial leverage.

Why recursive preferences?—the CRRA case. Recursive preferences are crucial to our mechanism because they allow for realistic dynamics of asset prices without compromising the model's performance with respect to macroeconomic quantities. The second data column of Table 13 reports the results obtained in the standard CRRA case. The dynamics of

Table 13: Simulated moments for macroeconomic quantities and asset prices. Reported values are averages of 1,000 simulations over 72 quarters. The subscripts d (domestic) and f (foreign) are suppressed when there is no ambiguity. 'Baseline' refers to our baseline calibration in Table 12. 'CRRA' refers to the case of constant relative risk aversion case and is obtained by setting $\psi = 1/\gamma$. 'EXO' refers to the model with exogenous growth. 'Fast Adoption' refers to a calibration with $\vartheta_d^f = 0.14/4$. Excess returns are obtained assuming a leverage of 3.

	Baseline	CRRA	EXO	Fast adoption
Macro Quantities:				
$\mathrm{E}(\Delta c)$	1.900	1.900	1.900	1.900
$\mathrm{Std}(\Delta c)$	2.130	2.354	2.468	2.132
$\mathrm{ACF}_1\ \mathrm{E}_t(\Delta c_{t+1})$	0.888	0.896	0.922	0.888
$\mathrm{Std}\ \mathrm{E}_t(\Delta c_{t+1})$	0.084	0.022	0.061	0.087
$\mathrm{E}(\Delta y)$	1.900	1.900	1.900	1.900
$\mathrm{Std}(\Delta y)$	2.253	2.253	2.272	2.258
$\mathrm{E}(\Delta z)$	1.900	1.900	1.900	1.900
$\mathrm{Std}(\Delta z)$	3.474	3.473	3.503	3.483
$\mathrm{ACF}_1\ \mathrm{E}_t(\Delta z_{t+1})$	0.917	0.977	0.912	0.919
$\mathrm{Corr}(\Delta c_d, \Delta c_f)$	0.322	0.401	0.343	0.294
$\mathrm{Corr}(\Delta y_d, \Delta y_f)$	0.352	0.363	0.348	0.343
$\mathrm{Corr}(\Delta z_d, \Delta z_f)$	0.353	0.363	0.348	0.343
Innovation:				
$\mathrm{Std}(\Delta s)$	2.962	2.380	n.a.	2.987
ACF_1 (S/N)	0.903	0.910	n.a.	0.904
Corr $(S_d/N_d, S_f/N_f)$	0.300	0.148	n.a.	0.314
Asset prices:				
$\mathrm{E}(r^f)$	2.227	19.770	2.680	2.236
$\mathrm{Std}(r^f)$	0.050	0.214	0.037	0.052
$\mathrm{ACF}_1\ (r^f)$	0.888	0.896	0.922	0.888
$\mathrm{Std}(\Delta q)$	9.718	25.795	5.920	8.796
$\mathrm{Std}(m)/\mathrm{E}(m)$	29.328	23.638	5.662	29.505
$\mathrm{Corr}(m_d, m_f)$	0.944	0.401	0.453	0.954
$\mathrm{Corr}(m_d^{SR}, m_f^{SR})$	0.322	0.401	0.343	0.294
$\mathrm{Std}(m^{SR})$	1.278	23.544	1.481	1.279
$\mathrm{Corr}(m_d^{LR}, m_f^{LR})$	0.956	n.a.	0.490	0.966
$\mathrm{Std}(m^{LR})$	28.004	n.a	4.186	28.180
$\mathrm{E}(r^{\mathrm{s}}) - r^f$	2.009	1.097	0.186	1.981
$\mathrm{Std}(r^{\mathrm{s}} - r^f)$	7.116	4.554	5.061	7.130
$\mathrm{Corr}(r_d^{\mathrm{s}}, r_f^{\mathrm{s}})$	0.656	0.218	0.363	0.744

macroeconomic quantities within each country are only marginally changed in the CRRA case from the baseline calibration, but the reported cross-country moments reveal that our model with CRRA preferences has serious shortcoming: it cannot account for the sizable wedge observed in the data between cross-country correlations in consumption growth and stock market returns. The cross-country correlation in consumption growth is too high (0.40), and the cross-country correlation in stock market returns is too low (0.218). Also, notice that the cross country correlation in R&D expenditure drops to 0.148 because a positive productivity shock in one country has significantly less impact on the prospects of future growth of the other country than in the case of recursive preferences. Finally, the average of the risk-free rate is too high (19.77%)—a manifestation of the well-known risk-free rate puzzle.

Why endogenous growth?–the EXO case. When growth is exogenous, the risk propagation mechanism at the core of our baseline calibration is muted. Put simply: short-run risk, which results from the exogenous productivity process, does not have significant long-run effects on growth. In this case, the power of recursive preferences is limited. Even though recursive preferences makes agents fear variations in future utility, such variation is too small to have significant quantitative effects on equilibrium asset prices. Indeed, the volatility of the long-run component of the stochastic discount factor is only about 4%, or roughly 7 times smaller than what is obtained from the baseline model with endogenous growth. Similarly, the Sharpe Ratio declines to 0.06, and both the volatility of the exchange rate (5.92%) and the cross-country correlation in stock market returns (0.363) are lower than their empirical counterparts.

The role of international adoption—the fast adoption case. The last column of Table 13 shows the results we obtain after increasing the ϑ_d^f parameter to 0.144/4. This value remains within the range of our empirical estimates yet implies foreign innovations are adopted more rapidly. According to our mechanism, faster adoption of foreign R&D has a significant effect on the dynamics of international asset prices. We see in particular that, relative to our baseline calibration, stock market returns are more correlated and exchange rates are less volatile in the fast adoption scenario. And in line with our model's risk-sharing mechanism (induced by recursive preferences), faster adoption generates more variation in the economy's future prospects—further increasing the difference between the cross-country correlation of consumption growth and that of stock market returns.

6 Conclusion

In this paper we offer a quantitative analysis of a symmetric, two-country, endogenous growth model of innovation and international adoption of foreign innovations through trade in varieties. We have shown—both theoretically and by way of a calibration exercise—that the joint presence of endogenous growth and recursive preferences has a significant impact on the equilibrium dynamics of both quantities and asset prices. In particular, we have seen that when both ingredients are present, the risk sharing mechanism in the model results in a significant wedge between the cross-country correlation of quantities and the cross-country correlation of asset prices.

We provide novel empirical evidence of our proposed mechanism. In particular, we show that country pairs with a higher R&D content of international trade have more highly correlated stock market returns and less volatile exchange rates. We show that these results are robust to the inclusion of various measures of trade intensity across countries, suggesting that R&D plays a crucial role in driving the cross-country equilibrium dynamics of asset prices.

The model developed in this paper could be extended to address other international asset-pricing puzzles. For instance, relaxing the symmetry assumption would allow one to analyze whether our mechanism might help explain observed deviations in the uncovered interest parity condition or the profitability of the currency carry trade. We leave these issues for future research.

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Appendix

A Trade Data, Asset Prices, and Comovements

In this appendix, we describe the relevant trade data and asset prices, after which we construct the measures used in our analysis.

Trade Data

The source of our trade data is UN COMTRADE. We collect product data at the 6-digit level of disaggregation; these data are annual and cover the period 1996-2013. Our focus is on the trade that occurs between importer i (identified by its IISCODE) and exporter j (identified by its EISCODE). We therefore collect data on the product types being traded (using their 6-digits identifiers it) and the dollar value of the trade in each product (i.e., the per-product trade value).

To derive our preliminary statistics, we calculate the fraction of world trade and world GDP accounted for by the countries in our sample (those countries are listed in footnote 5).

From this data, we construct the following measures:

- Trade Intensity (i, j): $TI_{i,j}$, the sum of the trade value of all traded products
- Extensive margin (i, j): $EM_{i,j}$, the number (the"count") of different types of goods imported by country i from country j
- Intensive Margin (i, j): $IM_{i,j}$, "how much", in US dollars, country i is trading on average for each product imported from country j

In order to compare these numbers across pairs of countries, we normalize them while accounting for each country's GDP. These normalized measures are defined as

$$\tilde{TI}_{i,j} = \frac{TI_{i,j} + TI_{j,i}}{GDP_i + GDP_j}$$

Each country pair is ordered: i is the importer and j is the exporter, i.e., $TI_{i,j}$ will usually be different from $TI_{j,i}$

Aside: we want to make sure that the relationship $\tilde{TI}_{i,j} = \tilde{EM}_{i,j}\tilde{IM}_{i,j}$ holds, so that, taking logs, we can easily run linear regressions.

Asset Prices

We consider two main statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the currency depreciation rate. We start from monthly observations of stock market returns (from Ken French's website) and exchange rates (from Global Financial Data). Then, using twelve monthly observation for each year of our sample, we construct the following measures:

Stock Market

- Annual cross-country stock market return correlations between country i and country j, $corr(r^s_{i,t}, r^s_{j,t})$, for each year t.

Exchange Rate

- Annual volatility of currency i depreciation rate with respect to currency j, $vol(\Delta q^j_{i,t})$, for each year t. The log depreciation rate for currency i with respect to currency j is defined as $\Delta q^j_{i,t} = q^j_{i,t} - q^j_{i,t-1}$, where $q^j_{i,t}$ is the log exchange rate level at time t for country i (in units of currency i per one unit currency j.

B Model Equations

This appendix consists of all the equations used to characterize the domestic economy. The foreign economy is represented by a set of analogous equations.

Preferences

$$U_{d,t} = \left\{(1-\beta)C^{\theta}_{d,t} + \beta \left(E_t \left(U^{1-\gamma}_{d,t+1}\right)\right)^{\frac{\theta}{1-\gamma}}\right\}^{\frac{1}{\theta}}$$

Stochastic discount factor

$$M_{d,t+1} = \beta \left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1} \left(\frac{U_{d,t+1}}{E_t \left(U^{1-\gamma}_{d,t+1}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma-\theta}$$

Final producers

$$Y_{d,t} = (Z_{d,t}L_{d,t})^{1-\alpha} K^{\alpha}_{d,t}$$

Labor

$$L_{d,t} = 1$$

Aggregate productivity

$$Z_{d,t} \equiv \Omega_{d,t} \left(\tilde{A}\right)^{\frac{1}{1-\alpha}} \left[hN_{d,t}^{d} + \left(\frac{h}{1-h}Q_t\right)^{\frac{\nu}{\nu-1}} N_{f,t}^{d}\right]$$

$$\tilde{A} = (\xi\nu)^{\frac{\xi}{1-\xi}}$$

$$\Omega_{d,t} = e^{a_{d,t}}$$

$$a_{d,t} = \varphi \ a_{d,t-1} + \varepsilon_{d,t}$$

First order condition of labor

$$W_{d,t} = (1-\alpha)(1-\xi)\frac{Y_{d,t}}{L_{d,t}}$$

First order condition of investment

$$q_{d,t} = \frac{1}{\Lambda'_{d,t}}$$

$$1 = E_t \left[M_{d,t+1} \left\{\frac{1}{q_{d,t}} \left(\alpha(1-\xi)\frac{Y_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}(1-\delta) - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\Lambda_{d,t+1}\right)\right\}\right]$$

Law of motion of capital

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda_{d,t}K_{d,t}$$

Investment adjustment costs

$$\Lambda_{d,t} \equiv \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right) = \frac{\alpha_1}{\zeta}\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta} + \alpha_2$$

$$\Lambda'_{d,t} = \alpha_1 \left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta-1}$$

Demand for domestic intermediate goods

$$X_{d,t}^d = (h\xi\nu Y_{d,t}G_{d,t}^{-\nu})^{\frac{1}{1-\nu}}$$

Demand for foreign intermediate goods (imports)

$$X_{f,t}^d = (Q_t^{-1}\nu Y_{d,t}G_{d,t}^{-\nu}\xi(1-h))^{\frac{1}{1-\nu}} = X_{d,i,t}^d \left(Q_t \frac{h}{1-h}\right)^{\frac{1}{\nu-1}}$$

Materials (intermediate goods)

$$G_{d,t} = \xi\nu Y_{d,t}\left[hN_{d,t}^d + \left(\frac{h}{1-h}Q_t\right)^{\frac{\nu}{\nu-1}} N_{f,t}^d\right]^{\frac{1-\nu}{\nu}}$$

Profits of intermediate producers

$$\Pi_{d,t}N_{d,t}^d = \pi_{d,t}^d N_{d,t}^d + \pi_{d,t}^f N_{d,t}^f$$

Profits of domestic producers in the domestic market

$$\pi_{d,t}^d = \left(\frac{1}{\nu} - 1\right) X_{d,t}^d$$

Profits of domestic producers in the foreign market

$$\pi_{d,t}^f = \left(\frac{1}{\nu} - 1\right) X_{d,t}^f$$

Present Discounted Value (PDV) of a domestic producers selling in the domestic market

$$V_{d,t}^d = \pi_{d,t}^d + (1-\phi)E_t[M_{d,t+1}V_{d,t+1}^d]$$

PDV of a domestic producers selling in the domestic market

$$V_{d,t}^f = \pi_{d,t}^f + (1-\phi)E_t[M_{d,t+1}V_{d,t+1}^f]$$

PDV of a domestic producers not-yet selling in the domestic market

$$J_{d,t}^f = (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta_d^f \pi_{dt+1}^f + (1-\vartheta_d^f)J_{d,t+1}^f\right)\right]$$

Value of an innovation

$$V_{d,t} = V_{d,t}^d + J_{d,t}^f$$

Law of motion of new technologies

$$N^d_{d,t+1} = \vartheta_{d,t} S_{d,t} + (1-\phi) N^d_{d,t}$$

$$\vartheta_{d,t} = \frac{\chi (N^d_{d,t} + \iota N^d_{f,t})}{S^{1-\eta}_{d,t} (N^d_{d,t})^{\eta}}$$

Free entry condition of innovation

$$S_{dt} = E_t \left[M_{d,t+1} V_{d,t+1} \right] \left(N^d_{d,t+1} - (1-\phi) N^d_{d,t} \right)$$

Law of motion of adopted technologies

$$N^f_{d,t+1} = \vartheta^f_d (1-\phi)(N^d_{d,t} - N^f_{d,t}) + (1-\phi) N^f_{d,t}$$

Resource Constraint

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N^d_{d,t} X^d_{d,t} + N^f_{d,t} X^f_{dt}$$

International risk sharing

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{f,t+1}}{M_{d,t+1}}$$

C Deriving the Stock Market

Dividends are generated by the final producers, the intermediate producers, and the innovators. The stock market is the present discounted value (PDV) of the future dividends generated by all the firms in the economy. Optimality implies the following asset-pricing condition:

$$\mathcal{P}_{d,t} \quad = \quad E_t [M_{d,t+1} (\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})] \quad ,$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price, and $\mathcal{D}_{d,t}$ is the aggregate market dividend. Substituting forward, we have

$$\mathcal{P}_{d,t} = E_t \sum_{i=0}^{\infty} M_{d,t+i+1} \mathcal{D}_{d,t+i+1}$$

Total dividends are

$$\mathcal{D}_{d,t} = D_{d,t} + N^d_{d,t} \pi^d_{d,t} + N^d_{f,t} \pi^d_{f,t} - S_{d,t} \quad ,$$

with the dividends of the final producers, $D_{d,t}$, evolving according to

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - N^d_{d,t}P^d_{d,t}X^d_{d,t} - N^d_{f,t}P^d_{f,t}X^d_{f,t} \quad .$$

Consider the PDV of the dividends of the final producers, $\mathcal{P}^{tan}_{d,t}$. We have

$$\mathcal{P}^{tan}_{d,t} = E_t\left[\sum_{i=0}^{\infty} M_{d,t+i+1}D_{d,t+i+1}\right]$$

or, in recursive form, $\mathcal{P}^{tan}_{d,t} = E_t\left[M_{d,t+1}\left(\mathcal{P}^{tan}_{d,t+1} + D_{d,t+1}\right)\right]$.

Result. $\mathcal{P}^{tan}_{d,t} = q_{d,t}K_{d,t+1}$.

Proof. Consider the following expression:

$$E_t(M_{d,t+1}D_{d,t+1}) = E_t\left[M_{d,t+1}\left(Y_{d,t+1} - I_{d,t+1} - W_{d,t+1}L_{d,t+1} - N^d_{d,t+1}P^d_{d.t+1}X^d_{d,t+1} - N^d_{f,t+1}P^d_{f,t+1}X^d_{f,t+1}\right)\right] \quad .$$

From the first-order condition (FOC) of labor it follows that

$$W_{d,t+1}L_{d,t+1} = (1-\alpha)(1-\varepsilon)Y_{d,t+1} \quad .$$

We can use the FOC for intermediate producers to rewrite the expression

$$N^d_{d,t}P^d_{dt}X^d_{d,t} + N^d_{f,t}P^d_{f,t}X^d_{f,t}$$

or, substituting for the prices of intermediate goods,

$$N^d_{d,t}\frac{1}{\nu}X^d_{d,t} + N^d_{f,t}\frac{1}{\nu}Q_tX^d_{f,t} \quad .$$

Now using

$$X^d_{f,t} = X^d_{d,t}\left(Q_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}},$$

we obtain

$$\left(N^d_{d,t}\frac{1}{\nu} + N^d_{f,t}\frac{1}{\nu}Q_t\left(Q_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t}(Q_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)\frac{1}{\nu}X^d_{d,t} \quad .$$

Similarly, using

$$X^d_{d,t} = \left(h\xi\nu Y_{d,t}G^{-\nu}_{d,t}\right)^{\frac{1}{1-\nu}}$$

and substituting $G_{d,t} = \xi\nu Y_{d,t}\left[hN^d_{d,t} + \left(\frac{h}{1-h}Q_t\right)^{\frac{\nu}{\nu-1}}N^d_{f,t}\right]^{\frac{1-\nu}{\nu}}$, we have

$$X^d_{d,t} = \xi\nu Y_{d,t}\left[hN^d_{d,t} + \left(\frac{h}{1-h}Q_t\right)^{\frac{\nu}{\nu-1}}N^d_{f,t}\right]^{-1} \quad .$$

Plugging this expression into the total spending for intermediate producers, we have

$$\left(N^d_{d,t}\frac{1}{\nu} + N^d_{f,t}\frac{1}{\nu}Q_t\left(Q_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t}(Q_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)\frac{1}{\nu}X^d_{d,t} =$$

$$= \left(N^d_{d,t} + N^d_{f,t}(Q_t)^{\frac{\nu}{\nu-1}} \left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right) \frac{1}{\nu}\varepsilon\nu Y_{d,t} \left(N^d_{d,t} + N^d_{f,t}(Q_t)^{\frac{\nu}{\nu-1}} \left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)^{-1} = \varepsilon Y_{d,t} \quad .$$

Finally, consider the FOC for investment and rearrange it to obtain

$$q_{d,t}K_{d,t+1} = E_t\left[M_{d,t+1}\left(\alpha(1-\varepsilon)Y_{d,t+1} - I_{d,t+1}\right)\right] + E_t\left[M_{d,t+1}q_{d,t+1}\left((1-\delta)+\Lambda_{d,t+1}\right)K_{d,t+1}\right] \quad .$$

From the law of motion for capital, it follows that

$$\left(\frac{(1-\delta)+\Lambda_{d,t+1}}{K_{d,t+1}}\right) = \frac{K_{d,t+2}}{K_{d,t+1}}$$

substituting into the previous expression, then yields

$$q_{d,t}K_{d,t+1} = E_t\left[M_{d,t+1}\left(\alpha(1-\varepsilon)Y_{d,t+1} - I_{d,t+1}\right)\right] + E_t\left[M_{d,t+1}q_{d,t+1}K_{d,t+2}\right] \quad .$$

Letting $\hat{q}_t = q_t K_{t+1}$, solving the expression just displayed recursively, and imposing the standard transversality condition, we have

$$\hat{q}_t = E_t\left[\sum_{i=0}^{\infty} M_{t+i+1}\left(\alpha(1-\varepsilon)Y_{t+i+1} - I_{t+i+1}\right)\right] \quad .$$

Combining all these results, we finally obtain

$$\begin{aligned} \mathcal{P}^{tan}_{d,t} &= E_t\left[M_{d,t+1}\left(\mathcal{P}^{tan}_{d,t+1} + D_{d,t+1}\right)\right] \\ &= E_t\left[\sum_{i=0}^{\infty} M_{d,t+i+1}\left(-I_{d,t+i+1} + (\alpha)(1-\varepsilon)Y_{d,t+i+1}\right)\right] = \hat{q}_{d,t} \end{aligned}$$

□

We now calculate the present discounted values of the remaining terms in the expression for the market dividends. When computing the PDV of the profits of all the existing intermediate producers, we need to account for the existence of two types of intermediate producers in the economy: firms currently selling to both the domestic and the foreign market; and firms that are not yet selling to the foreign market but that might do so in the future.

We start by calculating the PDV of each firm that currently sells to both the domestic and foreign market today. These firms continue selling to both markets until they disappear disappear with probability ϕ. Let's denote the present discounted value of the dividends of these firms as $\pi_{d,t}$, which are given in recursive form by

$$\pi_{d,t} = \pi^d_{d,t} + \pi^f_{d,t} + (1-\phi)E_t\left[M_{d,t+1}\pi_{d,t+1}\right] = V^d_{d,t} + V^f_{d,t}$$

where the last equality uses the definition of the value function for one firm selling only to the domestic market and the value function of one firm that is already selling to the foreign market. From the previous expression, the expected PDV of the future dividends for one firm that sells both in the domestic and in the foreign market, which is the component that we need to compute the stock market and we call V_{1t}, is

$$V_{1t} = (1-\phi)E_t\left[M_{d,t+1}\pi_{d,t+1}\right] = \left(V^d_{d,t} - \pi^d_{d,t}\right) + \left(V^f_{d,t} - \pi^f_{d,t}\right)$$

Finally, the PDV of the dividends of firms that only sell today in the domestic market but have a chance to sell tomorrow to the export market is given, in recursive form, by

$$\pi^d_{d,t} + (1-\phi)E_t\left[M_{d,t+1}\left(\pi^d_{d,t+1} + J^f_{d,t+1}\right)\right] = V^d_{d,t} + (1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

From the previous expression, the expected PDV of the future dividends for a firm that sells only in the domestic market, which is the component that we need to compute the stock market and we call V_{2t}, is

$$V_{2t} = (1-\phi)E_t\left[M_{d,t+1}\left(\pi^d_{d,t+1} + J^f_{d,t+1}\right)\right] = \left(v^d_{d,t} - \pi^d_{d,t}\right) + (1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

Since there are $N^f_{d,t}$firms selling to both the domestic and the foreign market and $\left(N^d_{d,t} - N^f_{d,t}\right)$ firms selling only to the foreign market, we can aggregate the previous expressions to obtain the component of the stock market that is driven by the already established intermediate producers as

$$N^f_{d,t}V_{1t} + (N^d_{d,t} - N^f_{d,t})V_{2t} =$$

$$= N^f_{d,t}\left[\left(V^d_{d,t} - \pi^d_{d,t}\right) + \left(V^f_{d,t} - \pi^f_{d,t}\right)\right] + (N^d_{d,t} - N^f_{d,t})\left[\left(V^d_{d,t} - \pi^d_{d,t}\right) + (1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]\right]$$
$$= N^d_{d,t}\left(V^d_{d,t} - \pi^d_{d,t}\right) + N^f_{d,t}\left(V^f_{d,t} - \pi^f_{d,t}\right) + (N^d_{d,t} - N^f_{d,t})\left((1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]\right)$$

Therefore, there are three components to the stock market

1. Price of installed capital
$$q_{d,t}K_{d,t+1}$$
2. Value of intangible capital
$$N^d_{d,t}\left(V^d_{d,t} - \pi^d_{d,t}\right) + N^f_{d,t}\left(V^f_{d,t} - \pi^f_{d,t}\right)$$
3. Value of intangible capital that could possibly be sold abroad
$$(1-\phi)(N^d_{d,t+1} - N^f_{d,t+1})E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

D Additional Empirical Results

In this appendix, we report results–based on 5-year overlapping measures of asset prices—regarding the relationship between asset prices and the R&D content of international trade. These results are consistent with those obtained in Section 2.

Table 14: R&D content of international trade and stock market returns (5 years)

	(1)	(2)
$\log(X_{ij}^{\mathrm{R\&D}})$	0.04153***	0.03543***
	(0.001)	(0.001)
Time fixed effects	No	Yes
Observations	2660	2660
R^2	0.247	0.462

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 15: R&D content of international trade and exchange rate volatility (5 years)

	(1)	(2)
$\log(X_{ij}^{\mathrm{R\&D}})$	-0.00223***	-0.00261***
	(0.000)	(0.000)
Time fixed effects	No	Yes
Observations	770	770
R^2	0.029	0.183

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 16: R&D content of international trade(industry) and stock market returns (5 years)

	(1)	(2)
log($X_{ij}^{\mathrm{R\&D}}$)	0.03713***	0.03612***
	(0.004)	(0.004)
Time fixed effects	No	Yes
Observations	306	306
R^2	0.206	0.296

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 17: R&D content of international trade(industry) and exchange rate volatility (5 years)

	(1)	(2)
log($X_{ij}^{R\&D}$)	-0.00176	-0.00170
	(0.001)	(0.001)
Time fixed effects	No	Yes
Observations	90	90
R^2	0.034	0.078

Note: Standard errors are reported in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$