International R&D Spillovers and Asset Prices

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Abstract

We study the international propagation of long-run risk in the context of a general equilibrium model with endogenous growth. Innovation and international diffusion of technologies are the channels at the core of our mechanism. A calibrated version of the model matches several asset pricing and macroeconomic quantity moments, alleviating some of the puzzles highlighted in the international macro-finance literature. Our model predicts that country pairs that share more R&D have less volatile exchange rates and more correlated stock market returns. Using data from a sample of 19 developed countries, we provide suggestive empirical evidence in favor of our model's predictions.

Keywords: innovation, international diffusion, trade, asset prices, recursive preferences.

JEL classification: F3, F4, O3

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1 Introduction

The cross-country correlation in aggregate consumption growth is low. Sharpe ratio estimates of the stock market are large implying substantial variability in stochastic discount factors. These two simple facts represent a significant challenge to international equilibrium economic models with complete financial markets. Aggregate consumption growth is too smooth and does not covary enough with stock returns. On the other hand, in a frictionless economy, agents should share risk perfectly and equalize their marginal rates of substitution: the cross-country correlation in consumption growth should be equal to one. In the data, it is almost always below 0.4 (see, among others, Brandt, Cochrane, and Santa-Clara 2006).

Starting with the seminal paper of Bansal and Yaron (2004), the long-run risk finance literature has proposed a solution to this puzzling empirical evidence. The explanation relies on investors fearing variation in the future growth prospects of the economy. The key ingredient is a small predictable component–the long-run risk–with two crucial features: i) high correlation *within* a country and ii) high correlation *across* countries. In these models, long-run risk is *exogenously* specified. But where does it come from and why does it have such distinct dynamics? In this paper, we present a general equilibrium model that *endogenously* generates long-run risk with the required characteristics, with an emphasis on its international aspects.

We build a two-country endogenous growth model of innovation and international technology diffusion through trade in varieties. Growth in each country is driven by the accumulation of technology through endogenous innovation. Innovation is the result of research and development (R&D) as in Comin and Gertler (2006) and Kung and Schmid (2015). We posit that the technology embodied in intermediate goods spreads across countries via international trade. As a result, the aggregate expected productivity of a country depends not only on its own innovation but also on the foreign innovations embodied in imported intermediate products. Preferences are recursive so that, when agents have preference for the early resolution of uncertainty, they fear variation in the economy's long-run future growth prospects. We assume that international financial markets are complete. Endogenous innovation, together with recursive preferences, makes the equilibrium growth path risky due to its effect on the present discounted value of future profits of all firms in the economy.

Our endogenous growth mechanism works as follows. Within countries, R&D drives a small and persistent component of equilibrium growth rates. International diffusion renders this component common across countries. The intuition is that a technology shock in the home country affects the incentives to innovate not only in that country but also abroad, and in turn affects the prospects of global growth. The final effect is that a short-run technology shock in the home country has a long-run impact on the dynamics of future growth rates in both the home and the foreign economy. The effect is stronger for pairs of countries that share more technological innovations.

If preferences are recursive, the variations in the economy's future prospects have a significant effect on asset prices and on the cross-country correlation of equilibrium economic quantities. As technological innovation diffuses across countries through trade in varieties, it generates a sizable common component in discount rates that drives up the correlation in stock market returns while reducing the volatility of exchange rates. Our model replicates these international asset-pricing facts together with a realistic calibration of other macroeconomic quantities. More specifically, the model predicts—in line with the data— that realized consumption growth is only somewhat correlated across countries. This result is a consequence of the international risk sharing mechanism implied by recursive preferences (Colacito and Croce 2013).

We calibrate the model to data on macroeconomic quantities and asset prices. The crucial parameter is the speed of international diffusion of R&D. It governs the probability that a novel technology developed in the home country is adopted by the foreign country. We carefully calibrate this parameter using disaggregated bilateral trade data and R&D expenditures for a panel of 19 developed countries and the 1996-2013 period. The value we find implies that the average country adopts a new foreign technology every 6 years. In our quantitative results, we find that this modest spillover effect is sufficient to generate substantial comovement in the future growth prospects of the economy. Long-run risk becomes common across countries thanks to the international diffusion of R&D efforts. Our calibrated model predicts that country-pairs that share more R&D, that is those pairs with a faster speed of adoption, have more correlated stock market returns and less volatile exchange rates. In an empirical section at the end of the paper, we provide suggestive empirical evidence in favor of these model's predictions. Finally, our model can deliver sensible dynamics for Sharpe ratios and the risk-free rate across countries.

Our paper is related to several strands of the literature. First, the macroeconomic mechanism is related to the literature on endogenous growth through innovation. Technological innovation is a fundamental source of sustained economic growth (Romer 1990). Comin, Gertler, and Santacreu (2009) study the dynamics of the stock market in a model with endogenous growth. Kung and Schmid (2015) extend the one-country model to include recursive preferences.[1] We develop our model along those lines and extend it to an international setting in which technology diffuses across countries. One channel of diffusion that has received particular attention in the literature is the trade in products (varieties) that incorporate such technology so that a country's growth rate depends not only on its own innovation but also on the innovation efforts of its trading partners (Coe and Helpman 1995; Keller 1998).[2]

The second strand of the literature is related to papers that analyze the implications for asset pricing of the canonical "real business cycle model". Examples include habit-based models (Jermann 1998; Boldrin, Christiano, and Fisher 2001), studies on recursive preferences (Tallarini 2000; Campanale, Castro, and Clementi 2010; Kuehn 2007; Kaltenbrunner and Lochstoer 2010; and Papanikolaou 2011), and disaster risk analysis (Gourio 2012). Our paper differs from these by focusing on the international dimension of asset prices.

The third related strand of literature is that on technology adoption and innovation through international trade in varieties (e.g., Broda, Greenfield, and Weinstein 2006; Santacreu 2015). Using highly disaggregated trade data, this research finds that adopting foreign innovations–by trade in varieties–affects the home country's growth rate. However, these authors do not discuss the asset–pricing implications of their mechanism, which is one of the main contributions of our study.

Finally, our paper is related to the literature on asset pricing with long-run risk that began with the seminal one-country model of Bansal and Yaron (2004) and was then applied to the international setting by Bansal and Shaliastovich (2013), Colacito and Croce (2011), Colacito and Croce (2013), and Colacito, Croce, Ho, and Howard (2017). Whereas these papers specify global long-run risk exogenously, our model shows how such risk arises endogenously through innovation and the international diffusion of R&D. From a methodological standpoint, our

[1] Several papers have examined the link between technological growth and prices; examples include Pastor and Veronesi (2009), Garleanu, Panageas, and Yu (2012), and Garleanu, Kogan, and Panageas (2012). These papers assume that technology growth is *exogenous*. In contrast, and consistently with our empirical findings, we focus on the relation between international asset prices and *endogenous* growth through R&D.

[2] Other potential channels include multinational firms spreading the benefits of R&D across countries (Ramondo 2009; Guadalupe, Kuzmina, and Thomas 2012; Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple 2015; Cravino and Levchenko 2015) and knowledge spillovers by way of international networks (Cai and Li 2012). The analysis of these channels is beyond the scope of our paper.

paper is related to that of Croce, Nguyen, and Schmid (2013), who examine the role of fiscal policy in an international endogenous growth model featuring agents who are concerned about robustness. Unlike that paper, ours focuses on the empirical links among trade, R&D, and asset prices in the context of an endogenous growth model with recursive preferences.

The rest of our paper is organized as follows. Section 2 introduces a reduced-form model that captures our main mechanism of international diffusion of long-run risk. Section 3 describes our general equilibrium model of endogenous growth. In Section 4 we describe the main mechanism at work, and Section 5 presents the calibration and our quantitative analysis. Section 6 looks at the empirical evidence, and Section 7 concludes.

2 A Reduced-Form Model of Long-Run Risk Spillovers

Following the seminal paper of Bansal and Yaron (2004), the long-run risk literature has forcefully argued in favor of the existence of a small, predictable news component driving the future prospects of the economy. For this component to have important quantitative implications for asset prices it must be priced — hence the use of recursive preferences — but it also needs to be: i) highly autocorrelated *within* a country, and ii) highly correlated *across* countries. Where does long-run risk come from? Previous studies have investigated how, in the context of a one-country general equilibrium model, long-run risk can arise endogenously from agent's consumption and investment decisions. Our paper proposes a mechanism to understand the drivers of the high cross-country correlation of the long-run risk across countries: international technology diffusion. In this section, we describe a reduced-form model that captures our novel mechanism of international diffusion of long-run risk. In Section 3, we introduce our benchmark general equilibrium model, in which we endogenize such mechanism.

Let the home and foreign long-run risk be described by the following two-dimensional autoregressive process $x_t \equiv (x_t^H, x_t^F)'$:

$$x_t = \Phi x_{t-1} + \varepsilon_t \quad . \tag{1}$$

We restrict our attention to a symmetric calibration in which the autocorrelation matrix Φ

is given by

$$\Phi = \begin{pmatrix} \varphi & \varphi_H^F \\ \varphi_H^F & \varphi \end{pmatrix}$$

and the $\varepsilon \equiv (\varepsilon^H, \varepsilon^F)'$ innovations are *i.i.d.* and normally distributed as follows:

$$\begin{pmatrix} \varepsilon^H \\ \varepsilon^F \end{pmatrix} \sim \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \rho_\varepsilon \sigma_\varepsilon^2 \\ \rho_\varepsilon \sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{bmatrix} \right) \quad .$$

For stationarity, the eigenvalues of Φ must lie within the unit circle. To showcase the mechanism of our paper, we analyze two versions of equation (1), both of which generate the required correlation structure of long-run risk.

First, we consider a *univariate* long-run risk specification, in which we shut down the off-diagonal elements of the autocorrelation matrix Φ, by imposing $\varphi_H^F = 0$. In this case, lagged values of x^F (x^H) *do not* affect current values of x^H (x^F). With this restriction in place, it is straightforward to show that $Corr(x_t^H, x_{t-1}^H) = Corr(x_t^F, x_{t-1}^F) = \varphi$ and $Corr(x_t^H, x_t^F) = \rho_\varepsilon$. Therefore, for long-run risk to be highly correlated both *within* and *across* countries, it must be that $\varphi \approx +1$ and $\rho_\varepsilon \approx +1$. Many papers have studied the nature of the autocorrelation coefficient φ and its effect on asset prices (see, among others, Bansal and Yaron 2004 and Bansal, Kiku, and Yaron 2016). In this paper, we focus on the cross-country correlation and notice that in this univariate specification it can be as high as prescribed by the literature only if we assume a high cross-country correlation in the long-run innovations, ε^H and ε^F. This is the solution adopted by Bansal and Shaliastovich (2013) and Colacito and Croce (2011). In sum, for a univariate specification to work, one must assume both a high autocorrelation coefficient and a high cross-country correlation in long-run innovations.

The second specification we consider is a *bivariate* long-run risk process which is closer in spirit to the mechanism of our general equilibrium model. This specification features an international spillover effect, which we capture by allowing for non-zero off-diagonal elements of Φ ($\varphi_H^F \neq 0$). Therefore, lagged values of x^F (x^H) *do* affect current realizations of x^H (x^F). In order to isolate the effect of the interaction term on the correlation structure of home and foreign long-run risk, we shut down the cross-country correlation in the ε shocks and impose

$\rho_\varepsilon = 0$.[3] We ask the following question: How strong must the spillover effect be in order to obtain a high cross-country correlation between x^H and x^F? The answer is: *not much.* To see this, we conduct a simple sensitivity exercise in which we change, gradually, the intensity of the international spillover effect. What is striking is that a small international spillover is enough to generate a large cross-country correlation of long-run risk. For example, for values as small as $\varphi_H^F = 0.05$, the cross-country correlation is as high as 0.80.

While the magnitude of the spillover effects is hard to interpret in this reduced-form setting, in the next section we present a general equilibrium model in which they are tightly connected with the international diffusion of technologies, generating a strong comovement between home and foreign long-run growth prospects.

3 Model

In this section, we present a model of innovation and international technology diffusion in which long-run risk arises endogenously within a country and is shared internationally through the adoption of foreign technologies. Each country has a representative household, with recursive preferences, that consumes a final good. A final good producer uses labor, capital, and a composite of intermediate goods—which we call materials—to produce a non-traded final good that is used for consumption, investment in physical capital, and investment in R&D. Materials are produced using traded intermediate goods (varieties), both domestic and foreign, which are produced by monopolistic competitive firms. The production of materials reflects a love-of-variety effect: if expenditures are held constant, a higher number of varieties increases the country's aggregate productivity. New varieties are introduced in each country through an endogenous process of innovation, after which they spread exogenously across countries through a slow process of adoption. Endogenous innovation and exogenous adoption, along with recursive preferences, are the key features at the core of our mechanism. Financial markets are assumed to be internationally complete.

Next, we describe the domestic economy d; the foreign economy f is defined analogously. Throughout the paper, a variable's subscript refers to the origin country — the exporter — while its superscript refers to the destination country — the importer. Unless otherwise specified, we use upper case letters to denote variables in levels, lower case letters to denote

[3] This restriction is relaxed in the full model of Section 3.

variables in log, and $\Delta x_{t+1} = x_{t+1} - x_t$ to denote the log-growth of variable X between time t and $t+1$.

3.1 Households

The domestic representative household has Epstein and Zin (1989) recursive preferences over consumption:

$$U_{d,t} = \{(1-\beta)C_{d,t}^{\theta} + \beta(E_t[U_{d,t+1}^{1-\gamma}])^{\theta/(1-\gamma)}\}^{1/\theta}. \tag{2}$$

In this expression, U_d is utility, t denotes time, β is the subjective discount factor, C_d denotes consumption, E is the expectations operator, γ is the constant relative risk aversion (CRRA), $\theta = \frac{1-\gamma}{1-1/\psi}$, and $\psi \equiv \frac{1}{1-\theta}$ is the intertemporal elasticity of substitution (IES). We assume that $\psi > 1/\gamma$, so that the representative agent has a preference for early resolution of uncertainty and fears variations in the economy's long-run prospects.

The stochastic discount factor (SDF) is given by

$$M_{d,t+1} = \beta \left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1} \left(\frac{U_{d,t+1}}{E_t[U_{d,t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1-\gamma-\theta}. \tag{3}$$

The first term of the right-hand side, $\beta\left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1}$, captures short-run risk. We refer to it as the short-run component of the SDF, M_d^{SR}. The second term, $\left(\frac{U_{d,t+1}}{E_t[U_{d,t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1-\gamma-\theta}$, captures the agent's concerns about uncertainty in future growth. We refer to it as the long-run component of the SDF, M_d^{LR}. The household consumes, supplies labor to the final good producer, and makes investment and savings decisions while participating in complete international financial markets. Accordingly, her budget constraint is

$$C_{d,t} + E_t\Big[M_{d,t+1}\mathcal{A}_{d,t+1}\Big] = W_{d,t}L_{d,t} + \mathcal{A}_{d,t},$$

where $W_{d,t}$ is the wage rate, $L_{d,t}$ denotes hours worked, and $\mathcal{A}_{d,t}$ is the state-contingent value of the household's financial wealth. Since there is no disutility of labor, the household supplies its entire endowment, which is normalized to one.

3.2 Final Good Producer

Domestic final producers are perfectly competitive. They use capital ($K_{d,t}$), labor ($L_{d,t}$), and a composite of domestic and foreign intermediate goods, which we call materials ($G_{d,t}$), to produce a nontraded final good ($Y_{d,t}$) in accordance with the following Cobb–Douglas production function:

$$Y_{d,t} = \left(K_{d,t}^{\alpha}\left(\Omega_{d,t}L_{d,t}\right)^{(1-\alpha)}\right)^{(1-\xi)} G_{d,t}^{\xi}. \tag{4}$$

Materials $G_{d,t}$ are defined as

$$G_{d,t} = \left[h\left(\sum_{i=1}^{N_{d,t}^{d}}(X_{d,i,t}^{d})^{\nu}\right) + (1-h)\left(\sum_{i=1}^{N_{f,t}^{d}}(X_{f,i,t}^{d})^{\nu}\right)\right]^{1/\nu}; \tag{5}$$

Here $X_{d,i,t}^{d}$ (resp. $X_{f,i,t}^{d}$) is the amount of domestically (resp. foreign) produced intermediate good i that is used for final production in the domestic economy, $N_{d,t}^{d}$ (resp. $N_{f,t}^{d}$) is the mass of domestic (resp. foreign) intermediate goods that is used by domestic final producers, and $1/(1-\nu)$ is the elasticity of substitution across intermediate goods (with $\nu < 1$). Intermediate goods are aggregated according to a constant elasticity of substitution' production function (à la Ethier). The parameter h captures the degree of home bias. The parameter α represents the physical capital share, and ξ is the share of materials in final production.

The exogenous process $\Omega_{d,t}$ is the only source of exogenous uncertainty in our model. We assume that $\Omega_{d,t} = e^{a_{d,t}}$, where $a_{d,t}$ follows the following process:

$$a_{d,t} = \varphi\ a_{d,t-1} + \rho_{ec}(a_{f,t-1} - a_{d,t-1}) + \varepsilon_{d,t},$$

where $|\varphi| < 1$ and $\varepsilon_{d,t} \sim N(0, \sigma^2)$. We allow for cross-country correlation in the exogenous technology shocks and let $\rho = \mathrm{corr}(\varepsilon_{d,t}, \varepsilon_{f,t})$. The parameter $\rho_{ec} \in (0,1)$ is a small error correction term calibrated to generate moderate cointegration. It ensures the stability of our solution method, yet has virtually no effect on the results (see, for instance, Colacito and Croce 2013).

Final producers choose capital, labor, investment, and intermediate goods to maximize shareholder's value subject to the production technology (4). Formally, we have

$$\max_{\{I_{d,t}, L_{d,t}, K_{d,t+1}, X_{d,i,t}^{d}, X_{f,i,t}^{d}\}_{t\geq 0,}} E_0\left[\sum_{t=0}^{\infty} M_{d,t}D_{d,t}\right], \tag{6}$$

where the firm's dividends are given by

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - \sum_{i=1}^{N_{d,t}^{d}} P_{d,i,t}^{d} X_{d,i,t}^{d} - \sum_{i=1}^{N_{f,t}^{d}} P_{f,i,t}^{d} X_{f,i,t}^{d}. \tag{7}$$

Here $I_{d,t}$ is investment in physical capital, $P_{d,i,t}^{d}$ is the price of a domestically produced intermediate good, and $P_{f,i,t}^{d}$ is the price of a foreign-produced intermediate good that is used for domestic production. Both prices are expressed in units of the domestic producer's final good.

The law of motion for physical capital is given by

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right)K_{d,t}, \tag{8}$$

where $\delta \in (0,1)$ is the depreciation rate and where $\Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right)$ captures convex capital adjustment costs.[4]

3.3 Intermediate Good Producers

In each country, a set of monopolistic competitive firms produces a differentiated good using final output according to a constant returns to scale (CRS) production function, that is, one unit of the final output is used to produce one unit of the intermediate good. All intermediate producers produce with the same efficiency and for both the domestic and foreign market. Every period, each domestic intermediate producer i solves the following static profit maximization problem:

$$\begin{aligned}
\max_{P_{d,i,t}^{d}, P_{d,i,t}^{f*}} \Pi_{d,i,t} &\equiv \max_{P_{d,i,t}^{d}, P_{d,i,t}^{f*}} (\pi_{d,i,t}^{d} + \pi_{d,i,t}^{f}) \\
&= \max_{P_{d,i,t}^{d}} \quad P_{d,i,t}^{d} \quad X_{d,i,t}^{d}(P_{d,i,t}^{d}) - X_{d,i,t}^{d} \quad (P_{d,i,t}^{d}) \\
&+ \max_{P_{d,i,t}^{f*}} \underbrace{(P_{d,i,t}^{f}\mathcal{E}_t)}_{P_{d,i,t}^{f*}} X_{d,i,t}^{f}(\underbrace{P_{d,i,t}^{f}\mathcal{E}_t}_{P_{d,i,t}^{f*}}) - X_{d,i,t}^{f}(\underbrace{P_{d,i,t}^{f}\mathcal{E}_t}_{P_{d,i,t}^{f*}}),
\end{aligned} \tag{9}$$

[4]As in Jermann (1998), $\Lambda_{d,t} \equiv (I_{d,t}/K_{d,t}) = (\alpha_1/\zeta)(I_{d,t}/K_{d,t})^{\zeta} + \alpha_2$. The parameters α_1 and α_2 are chosen so that there are no adjustments costs in the steady state, and $1/(1-1/\zeta)$ is the elasticity of the investment rate with respect to Tobin's Q.

where $\pi^{d}_{d,i,t}$ (resp. $\pi^{f}_{d,i,t}$) are the profits from selling the domestic product at home (resp. abroad) and where $P^{f^*}_{d,i,t} = P^{f}_{d,i,t}\mathcal{E}_t$ is the price (in units of the domestic good) of a domestically produced intermediate good that is being exported. We use $\mathcal{E}_t$ to denote the real exchange rate, defined as the number of domestic final goods per unit of foreign final good.[5]

3.4 Innovation and Adoption

So far, we have described the production process of the economy, while assuming a given set of available technologies or intermediate goods. Yet the number of intermediate goods evolves over time according to an endogenous process of innovation and an exogenous process of adoption. New technologies are introduced through innovation, and each new technology is then used to produce an intermediate good under monopolistic competition. Domestic intermediate goods can immediately be sold to the domestic final producer; however, for the good to be sold in a foreign market, it must be adopted first. In our model, adoption is equivalent to importing a foreign intermediate good. We assume this process to be slow and exogenous. Technology adoption has a positive effect on the country that adopts such technology, as it improves its final producer's productivity. Trade is the vehicle through which new technologies spread across countries and the motive for trade is given by the Armington aggregator in equation (5). The Armington model is based on the premise that each country produces a different good and final producers would like to consume at least some of each country's goods.

3.4.1 Innovation

In each country, innovators invest resources (final output) to introduce new prototypes of a product. A successful innovator starts producing the new good as an intermediate producer. Each domestic innovator i chooses $S_{d,i,t}$ units of final output to maximize the present discounted value of future profits that she expects to obtain from selling the good to both domestic and foreign producers.

[5] We express real prices of the intermediate goods in units of the importer's final good. This means that, when the domestic (resp. foreign) intermediate good is used for the production of the foreign (resp. domestic) final output, $P^{f}_{d,t} \equiv (1/\nu)\mathcal{E}_t^{-1}$ and $P^{d}_{f,t} \equiv (1/\nu)\mathcal{E}_t$.

The law of motion for new prototypes is

$$N^d_{d,t+1} = \vartheta_{d,t} S_{d,t} + (1-\phi) N^d_{d,t}; \tag{10}$$

where $N^d_{d,t}$ is the mass of new technologies arriving in country d at time t, ϕ is the exogenous probability that a new variety becomes obsolete, and $\vartheta_{d,t}$ is the productivity of innovation. Following Comin and Gertler (2006), we assume that this productivity takes the following functional form:

$$\vartheta_{d,t} = \frac{\chi N^d_{d,t}}{S^{1-\eta}_{d,t} (N^d_{d,t})^\eta}. \tag{11}$$

The process $S_{d,t} = \sum_{i=1}^{N^d_{d,t}} S_{d,i,t}$ represents total R&D expenditures in the domestic country (in terms of the domestic final good). The parameter η denotes the elasticity of innovation with respect to R&D, and χ is a scaling parameter. The term $N^d_{d,t}$ captures an externality on the innovation process of country d. It stems from the varieties that have already been invented by country d. In other words, an innovator can learn from its own past innovations. In this specification, $\vartheta_{d,t}$ is taken as given in the choice of optimal investment in R&D.

3.4.2 International Adoption

In every period, only a fraction of intermediate goods from country d can be imported by the final good producer of country f. The parameter ϑ^f_d, which governs the speed of adoption, is crucial in our mechanism. The law of motion for domestic intermediate goods that can be used by the foreign final producer evolves according to

$$N^f_{d,t+1} - (1-\phi) N^f_{d,t} = \vartheta^f_d (1-\phi)(N^d_{d,t+1} - N^f_{d,t}), \tag{12}$$

where $N^f_{d,t}$ is the number of domestic goods imported by the foreign economy. If follows that the mass of domestic varieties not yet adopted by the foreign country is equal to $N^d_{d,t+1} - N^f_{d,t}$. Adoption is not instantaneous but follows a slow moving process, as it has been documented by Keller (2004).[6]

[6] An alternative specification would be to endogenize the process of adoption by assuming a fixed cost of importing intermediate goods. In that case, firms would make a costly investment to be able to import the good, and the extensive margin (i.e., the number of intermediate products that are imported) would be endogenous. For simplicity, we assume adoption to be exogenous; every period, final producers import a

3.4.3 Value Functions

The value of a domestic innovation, $V_{d,i,t}$, is given by the present discounted value of the profits that innovator i expects to obtain from selling the good domestically and abroad. Let the value of the domestic innovations that are immediately sold to domestic (resp. foreign) final producers be $V^d_{d,i,t}$ (resp. $V^f_{d,i,t}$), and let the value of the domestic innovations that could be adopted by country f be $J^f_{d,i,t}$. Then

$$V_{d,i,t} = V^d_{d,i,t} + J^f_{d,i,t}, \tag{13}$$

where

$$\begin{aligned} V^d_{d,i,t} &= \pi^d_{d,i,t} + (1-\phi)E_t[M_{d,t+1}V^d_{d,i,t+1}], && (14) \\ V^f_{d,i,t} &= \pi^f_{d,i,t} + (1-\phi)E_t[M_{d,t+1}V^f_{d,i,t+1}], && (15) \\ J^f_{d,i,t} &= (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta^f_d V^f_{d,i,t+1} + (1-\vartheta^f_d)J^f_{d,i,t+1}\right)\right]. && (16) \end{aligned}$$

According to the value function (16), with probability ϑ^f_d the domestic firm can sell its product abroad at $t+1$, while with probability $1-\vartheta^f_d$ the product remains exclusively available for the domestic market.

Discounted future profits on innovations are the payoff to innovators. Because the R&D sector is competitive, the free-entry condition for R&D investment–in the symmetric equilibrium under which all firms are identical–is

$$S_{d,t} = E_t\left[M_{d,t+1}V_{d,t+1}\right]\left(N^d_{d,t+1} - (1-\phi)N^d_{d,t}\right) \tag{17}$$

or, equivalently,

$$\frac{1}{\vartheta_{d,t}} = E_t\left[M_{d,t+1}V_{d,t+1}\right]. \tag{18}$$

subset of the products that are produced in the rest of the world. Following Santacreu (2015), we calibrate this process to match the extensive margin of trade in the data.

3.5 Resource Constraint

Final output is used for consumption, investment in physical capital, intermediate goods production, and investment in R&D. Hence we may write the resource constraint as

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N_{d,t}^{d} X_{d,t}^{d} + N_{d,t}^{f} X_{d,t}^{f} \ .$$

3.6 Equilibrium and Steady State

We define a symmetric equilibrium as a set of equations according to which all firms within a country behave symmetrically.

A *general symmetric equilibrium* is defined as an exogenous stochastic sequence of technology shocks $\{\Omega_{d,t}, \Omega_{f,t}\}_{t=0}^{\infty}$, an initial vector $\{N_{d,0}^{d}, N_{d,0}^{f}, N_{f,0}^{f}, N_{f,0}^{d},, K_{d,0}, K_{f,0}\}$, a set of parameters $\{\beta, \theta, \gamma, \alpha, \xi, \varphi, \sigma, \rho, \delta, \zeta, \nu, \chi, \phi, \eta, h, \vartheta_{d}^{f}\}$, a sequence of aggregate prices $\{W_{d,t}, W_{f,t}, V_{d,t}, V_{f,t}, q_{dt}, q_{ft}, \mathcal{E}_{t}\}_{t=0}^{\infty}$, where q is the Tobin's Q, value functions $\{V_{dt}^{d}, V_{ft}^{f}, J_{dt}^{f}, J_{ft}^{d}, V_{dt}^{f}, V_{ft}^{d}\}_{t=0}^{\infty}$, a sequence of intermediate good prices $\{P_{d,t}^{d}, P_{d,t}^{f}, P_{f,t}^{f}, P_{f,t}^{d}\}_{t=0}^{\infty}$, a sequence of aggregate quantities $\{Y_{d,t}, Y_{f,t}, G_{d,t}, G_{f,t}, C_{d,t}, C_{f,t}, I_{d,t}, I_{f,t}, L_{d,t}, L_{f,t}, S_{d,t}, S_{f,t}\}_{t=0}^{\infty}$, quantities of intermediate goods $\{X_{d,t}^{d}, X_{d,t}^{f}, X_{f,t}^{f}, X_{f,t}^{d}\}_{t=0}^{\infty}$, a sequence of profits $\{\Pi_{d,t}, \Pi_{f,t}, \pi_{d,t}^{d}, \pi_{d,t}^{f}, \pi_{f,t}^{f}, \pi_{f,t}^{d}\}_{t=0}^{\infty}$, and laws of motion $\{N_{d,t+1}^{d}, N_{d,t+1}^{f}, N_{f,t+1}^{f}, N_{f,t+1}^{d}, K_{d,t+1}, K_{f,t+1}\}_{t=0}^{\infty}$ such that the following conditions hold:

(i) The state variables satisfy their respective laws of motion.

(ii) The endogenous variables solve the producers', innovators', and representative households' problems.

(iii) The resource constraint is satisfied.

(iv) Prices are such that all markets clear.

The equilibrium conditions are given in Appendix A.

3.7 Aggregate Productivity

Domestic aggregate productivity can be expressed as

$$Z_{d,t} \equiv \Omega_{d,t} \left(\tilde{A}\right)^{\frac{1}{1-\alpha}} \left[h N_{d,t}^{d} + \left(\frac{h}{1-h} \mathcal{E}_{t} \right)^{\frac{\nu}{\nu-1}} N_{f,t}^{d} \right], \tag{19}$$

where $\tilde{A} \equiv (\xi\nu)^{\xi/(1-\xi)}$. Taking logarithms, we have

$$\log Z_{d,t} = \log \Omega_{d,t} + \log \left\{ \left(\tilde{A}\right)^{1/(1-\alpha)} \left[hN^d_{d,t} + \left(\frac{h}{1-h} \mathcal{E}_t \right)^{\frac{\nu}{\nu-1}} N^d_{f,t} \right] \right\}.$$

Thus aggregate productivity has both an exogenous and an endogenous component, that is

$$\log(Z_{d,t}) = \log(Z^{EXO}_{d,t}) + \log(Z^{ENDO}_{d,t}).$$

where

$$\log(Z^{EXO}_{d,t}) \equiv \log \Omega_{d,t},$$

and

$$\log(Z^{ENDO}_{d,t}) \equiv \log \left\{ \left(\tilde{A}\right)^{1/(1-\alpha)} \left[hN^d_{d,t} + \left(\frac{h}{1-h} \mathcal{E}_t \right)^{\frac{\nu}{\nu-1}} N^d_{f,t} \right] \right\} \quad .$$

The exogenous component of aggregate productivity is given by the stochastic process $\Omega_{d,t}$. The endogenous component, which is the driver of growth in our model, depends on the number $N^d_{d,t}$ of varieties produced domestically, the number $N^d_{f,t}$ of varieties produced in the foreign country and that have been already adopted in the home country, and the real exchange rate, $\mathcal{E}_t$.

3.8 Dividends and the Stock Market

Stocks are viewed as claims on all the production sectors: the final good sector, the intermediate good sector, and the innovation sector. Hence the aggregate dividend is defined as the net payout from these production sectors; formally,

$$\mathcal{D}_{d,t} = D_{d,t} + N^d_{d,t}\pi^d_{d,t} + N^d_{f,t}\pi^d_{f,t} - S_{d,t}. \tag{20}$$

Optimality implies the following asset-pricing condition:

$$\mathcal{P}_{d,t} = E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})],$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price and $\mathcal{D}_{d,t}$ is the aggregate market dividend. In Appendix B, we show how to derive the price of the stock market as the present discounted value of all future dividends.[7]

3.9 Exchange Rate, and Risk Sharing

Since we assume that financial markets are complete, the exchange rate depreciation is given by the ratio of foreign to domestic stochastic discount factors. Thus,

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{M_{f,t+1}}{M_{d,t+1}}. \tag{21}$$

Because preferences are recursive, the risk-sharing mechanism is nonstandard. In particular, agents fear not only current shocks but also variation in future utility. Formally, let $\Upsilon_t = \mathcal{E}_t \left(\frac{C_{d,t}}{C_{f,t}}\right)^{\theta-1}$. Using the SDF from equation (3) together with the no-arbitrage condition (21), we can express Υ_t recursively as follows:

$$\Upsilon_{t+1} = \Upsilon_t \, \frac{M_{f,t+1}}{M_{d,t+1}} \, \frac{e^{(\theta-1)\Delta c_{d,t+1}}}{e^{(\theta-1)\Delta c_{f,t+1}}} \quad . \tag{22}$$

The variable Υ_t is constant in the CRRA case. Under Epstein–Zin recursive preferences, however, it evolves as a function of the cross-country realizations of agents' continuation utilities.

4 The Mechanism

We illustrate the model's main mechanism by analyzing the effect of an increase in domestic exogenous productivity on asset prices and quantities. After such a productivity increase, final producers demand more intermediate goods–both domestic and foreign. The resulting higher demand for intermediate goods increases the value of domestic innovations, with more resources being allocated to domestic R&D. In order to understand the effect on the foreign economy, we need to consider the international risk-sharing mechanism at work.

Recall that optimal R&D spending equals the present discount value of an innovation (see equation (17)). With recursive preferences the discount factor is volatile and highly correlated across countries. Growth is risky and a significant share of this risk is global. The result is a first order effect on the international correlation structure of stock market returns

[7] Comin, Gertler, and Santacreu (2009) derive a similar expression for a closed economy.

and on the volatility of exchange rates. Stock market returns are highly correlated across countries and exchange rates are relatively smooth.

While recursive preferences have a large effect on asset prices, their impact on realized consumption growth is limited. This is the combined result of two channels. The first is the well-known result that, in a standard real business cycle model, decoupling risk aversion from the elasticity of intertemporal substitution significantly improves a model's ability to replicate asset-pricing moments while leaving quantity moments virtually unaltered (see Tallarini 2000 and Kaltenbrunner and Lochstoer 2010). Unlike those papers, our model features endogenous growth, so that we observe *some* effects on quantities, as highlighted by Kung and Schmid (2015).

The second channel comes from complete markets. After a positive domestic productivity shock, the domestic economy becomes more innovative and expected future productivity increases. In turn, the long-run growth prospects of the economy improve. Endogenous growth transforms a short-run transitory shock into a long-run shock with persistent effects on future growth. With recursive preferences, the domestic consumer experiences a substantial drop in her marginal utility. These are very good times for the domestic economy, not only because of the current technological environment, but also because the future looks bright. As a result, domestic final producers demand more intermediate goods, both domestic and foreign. The higher demand for foreign intermediate goods induces agents in the foreign economy to divert resources from consumption towards investment in R&D. The outcome is a cross-country correlation in consumption that is lower than what we would obtain in the absence of our recursive risk-sharing mechanism. In contrast, stock market returns become even more highly correlated, since they reflect the present discounted value of future growth prospects. These prospects are shared across countries, as captured by the high cross-country correlation of the long-run components of the SDFs. In our model, the wedge in the dynamics between consumption and asset prices is driven mainly by expected future productivity growth. Indeed, consistently with previous studies, increases in realized current aggregate productivity have little effect on quantities (*e.g.*, Kehoe and Ruhl 2013).

The three ingredients highlighted in this section — endogenous innovation, international adoption and recursive preferences — are key for our results. In the next session, we prodive a quantitative analysis of their role in our model's mechanism.

5 Quantitative Implications

In this section, we discuss the quantitative implications of our model and explore its ability to replicate key international moments for macroeconomic quantities and asset prices. We put particular emphasis on the endogenous creation of long-run risk and its international diffusion through the adoption of foreign varieties. Our baseline model is calibrated at a quarterly frequency. We calibrate our model to the median country of our panel.

5.1 Calibration

We need to specify a total of 16 parameters, whose values are reported in Table 1. We start by discussing those that are relatively more standard. Values for the preference parameters are set in the spirit of the long-run risk literature (e.g., Bansal and Yaron 2004; Colacito and Croce 2013). In particular, we set the coefficient γ of relative risk aversion equal to 10 and the coefficient θ equal to 0.33, which implies an intertemporal elasticity of substitution of $\psi = 1.5$. Under this calibration of the preference parameters, agents in the economy fear shocks to expected future growth. The subjective discount factor is chosen to pin down the mean of the risk-free rate; thus, $\beta = 0.984^{1/4}$.

The parameters relating to the technology for final good production are obtained from Comin and Gertler (2006) and Kung and Schmid (2015). The capital share α is set to 0.35 to match the average capital share, and the share ξ of intangible capital is set to 0.5. The depreciation rate of physical capital is set to 0.02 to match the average capital investments rate and the parameter ζ is chosen to fix an elasticity of the investment rate with respect to Tobin's Q of 0.3. We follow the literature and use a value of 0.4 for ν; this parameter is related to the elasticity of substitution across intermediate goods and fixes the intermediate good markup in our model. The home bias parameter, h is set to 0.98, in line with the international estimates of Colacito and Croce (2013). We set the autocorrelation of the exogenous technology shock to $0.95^{1/4}$ and the volatility parameter σ at 1.84% in order to obtain a realistic volatility for consumption and output growth. Finally, we calibrate the cross-country correlation of the exogenous technology shocks, ρ, to 0.35, so that in equilibrium our model delivers a cross-country correlation of total factor productivity (TFP) growth of roughly 0.28, which is consistent with the empirical estimate of our sample of countries.

We now discuss the less standard parameters that govern the process of innovation and

Table 1: Parameters for the Baseline model.

Description	Parameter	Value
Preferences:		
Risk aversion	γ	10
IES	$\psi = 1/(1-\theta)$	1.5
Subjective discount factor	β^4	0.984
Final Production:		
Capital share	α	0.35
Share of materials	ξ	0.5
Autocorrelation of $\Omega = e^a$	φ^4	0.95
Volatility of exogenous shock ε	σ	1.84%
Cross-correlation of exogenous shock	ρ	0.35
Depreciation of capital stock	δ	0.02
Investment adjustment cost	$1/(1-1/\zeta)$	0.3
Inverse markup	ν	0.4
Innovation and International Adoption:		
Scale	χ	0.4240
Innovation obsolesce rate	ϕ	0.0375
Elasticity of innovation w.r.t. R&D	η	0.50
Home bias	h	0.98
International adoption	ϑ_d^f	0.04

technology adoption. The parameter χ is a pure scaling parameter; we set its value such that the steady-state growth rate of consumption has an annualized mean of 1.90%. Together with ν's value, χ then gives us a value of $\tilde{A} = (\xi\nu)^{\frac{\xi}{1-\xi}}$ that is consistent with balanced growth. The parameter η governs the elasticity of new varieties with respect to R&D and is set to 0.5, a number within the range of estimates given by Griliches (1990). Finally, we set $\phi = 0.0375$, which corresponds to a 15% annualized depreciation rate of the R&D stock. The last parameter we calibrate, ϑ_d^f, governs the speed of international adoption and is a key parameter in our model. We use annual data for 19 countries from 1996 to 2013 on R&D together with disaggregated bilateral trade data to obtain a value for this parameter. We thus have a panel of 190 unique country-pairs and 26 years.[8] For each unique country pair, we compute the extensive margin of trade, which is defined as the number of products traded among each country pair. We regress this variable against the average R&D stock of

[8] Details on the data and the estimation of this parameter are described in Section 6.

that country-pair, including year fixed effects. The coefficient derived from that regression captures the effect of a country-pair R&D expenditures on the rate at which its product can be adopted (equation (10)). Our estimates for this parameter range from 0.03 to 0.05, which imply that innovations take between 5 and 8 years to be adopted internationally. In our benchmark calibration, we use the average quarterly value of $\vartheta_d^f = 0.04$. This value implies that, consistently with the data, the share of traded varieties, N_d^f/N_d^d, is 0.36, and the steady-state R&D intensity, S/N, is 5%.

The model is solved using perturbation methods. We compute a third-order approximation of our policy functions using the Dynare++ package. All variables included in our code are expressed in log-units.[9]

5.2 Results

In this section, we describe the main results for the dynamics of macroeconomic quantities and asset prices. Later, in Section 5.3, we analyze the endogenous sources of long-run risk and conduct a sensitivity analysis of its international diffusion. Table 2 reports simulated moments of four different calibrations: Baseline, Fast Adoption, EXO and CRRA. For the Fast Adoption calibration, we increase ϑ_d^f by 20%. The EXO calibration corresponds to a model in which innovation is exogenous. In this version of the model, new prototypes arrive exogenously according to a Poisson process so that the steady-state growth rate of consumption does not change. For the CRRA calibration, we impose $\psi = 1/\gamma$ and leave all other parameters unaltered. Consistently with our empirical analysis, the length of our sample is 72 quarters. All results are averaged across 5,000 simulations. Table 2 contains the main moments of our four calibrations, together with their empirical counterparts.

Macroeconomic Quantities. Our Baseline model performs fairly well in terms of moments for both output and consumption growth. Their respective standard deviations are 2.27% and 2.44%. The model generates a cross-country correlation of consumption growth of 0.18. The relatively low correlation is in line with our empirical estimates and with previous studies. This value is lower than the calibrated cross-country correlation of exogenous

[9]For additional details on the solution and on the approximation of recursive economies with multiple agents see, Colacito and Croce (2012, 2013), Rabitsch, Stepanchuk, and Tsyrennikov (2015), and Colacito, Croce, Gavazzoni, and Ready (2017).

productivity shocks (0.35)—a consequence of the risk-sharing mechanism implied by recursive preferences. When a positive short-run shock hits the domestic economy, innovation and endogenous growth generate long-term growth effects that propagate to the rest of the world. Under recursive preferences, foreign marginal utility drops substantially and results in a reallocation of resources toward the production of foreign intermediate goods. The overall effect is a slight reduction in foreign consumption growth, which puts downward pressure in the cross-country correlation.

Our model performs quite well in terms of the dynamics of R&D expenditure. The volatility of R&D intensity, S/N, is 0.49% (0.53% in the data), the volatility of the growth rate of R&D intensity, $\Delta \log S/N$, is 2.18 (2.98 in the data) and its cross-country correlation is 0.21, also in line with the data (0.30). Innovation coming from R&D expenditure is the source of growth and its dynamics are crucial for the asset-pricing implications of our model.

Asset-pricing implications. The volatility of exchange rate depreciation is 8.62%. This result is a direct consequence of the joint presence of recursive preferences and a sizable amount of endogenous long-run growth uncertainty. The model generates a substantial amount of risk, as confirmed by its Sharpe ratio of 0.33, but SDFs are highly correlated across countries as a consequence of the international diffusion of long-run risk. Given such a large correlation in SDFs, stock market returns are also highly correlated across countries, with a correlation of 0.73, consistently with the empirical evidence.

The model generates a low and persistent risk-free rate, as observed in the data. However, as is common in international production models, its volatility is lower than what we observe in the data. Another limitation of our model, which is also common in the literature, is that the equity premium is low unless we assume the presence of financial leverage. Following Boldrin, Christiano, and Fisher (2001), we lever equity returns with a financial leverage of 5 and obtain $E(r^s - r^f) = 2.05$ and $\sigma(r^s - r^f) = 14.39$. Kung and Schmid (2015) and Colacito, Croce, Ho, and Howard (2012) adopt a similar solution.

The role of international adoption–Fast adoption case. When we increase ϑ_d^f to 0.047, long-run risk diffuses faster across countries. Faster adoption of foreign R&D has a mild impact on macroeconomic quantities, but a significant impact on the dynamics of international asset prices. Relative to our baseline calibration, stock market returns become more corre-

Table 2: Simulated moments for macroeconomic quantities and asset prices. Subscripts d (domestic) and f (foreign) are suppressed when there is no ambiguity. 'Baseline' refers to our baseline calibration in Table 1. 'Fast Adoption' refers to a calibration with $\vartheta_d^f = 0.47$. 'EXO' refers to the model with exogenous growth. 'CRRA' refers to the case of constant relative risk aversion case and is obtained by setting $\psi = 1/\gamma$. The risk premium, $r_m - r_f$, is levered following Boldrin, Christiano, and Fisher (2001). Data and standard error (s.e.) columns report median values for our 1996-2013 panel of countries. For *Innovation* variables we report moments of their low-frequency components, which are obtained using the bandpass filter from Christiano and Fitzgerald (2003) and isolating frequencies between 120 and 200 quarters. Means and standard deviations are reported as annualized percentages.

	Data	(s.e.)	Baseline	Fast adoption	EXO	CRRA
Macro Quantities:						
$Std(\Delta c)$	1.611	(0.140)	2.269	2.266	2.484	2.240
$Corr(\Delta c_d, \Delta c_f)$	0.192	(0.095)	0.178	0.184	0.41	0.731
$Std(\Delta y)$	2.236	(0.207)	2.441	2.439	2.350	2.241
$Corr(\Delta y_d, \Delta y_f)$	0.213	(0.103)	0.277	0.28	0.394	0.526
$Std(\Delta z)$	1.261	(0.225)	3.767	3.763	3.624	3.453
$Corr(\Delta z_d, \Delta z_f)$	0.373	(0.160)	0.277	0.280	0.394	0.526
Innovation:						
$Std(\Delta \log(S/N))$	2.187	(0.145)	2.983	2.983	n.a.	2.707
$Corr(\Delta \log(S_d/N_d), \Delta \log(S_f/N_f))$	0.207	(0.100)	0.303	0.302	n.a.	0.007
$Std(S/N)$	0.489	(0.041)	0.503	0.503	0	0.135
Asset prices:						
$E(r_f)$	2.020	(0.120)	2.273	2.275	2.793	19.98
$Std(r_f)$	1.229	(0.082)	0.195	0.095	0.082	0.415
$corr(r_{f,d}, r_{f,f})$	0.746	(0.039)	0.861	0.864	0.077	0.672
$Std(\Delta q)$	8.386	(1.006)	8.620	7.591	4.88	16.36
$E(r_m - r_f)$	6.050	(1.962)	2.018	2.058	0.518	1.303
$Std(r_m - r_f)$	24.33	(2.982)	14.39	13.40	9.108	6.010
$corr(r_{m,d} - r_{f,d}, r_{m,f} - r_{f,f})$	0.738	(0.096)	0.733	0.931	0.46	0.966
Long-run risk:						
$ACF_1\ E_t(\Delta c_{t+1})$			0.913	0.913	0.92	0.909
$StdE_t(\Delta c_{t+1})$			0.074	0.074	0.061	0.023
$CorrE_t(\Delta c_{d,t+1}), E_t(\Delta c_{f,t+1})$			0.765	0.775	0.081	0.395
$ACF_1\ E_t(\Delta z_{t+1})$			0.915	0.917	0.909	0.905
$Std(E_t(\Delta z_{t+1}))$			0.029	0.029	0.137	0.011
$Corr(E_t(\Delta z_{d,t+1}), E_t(\Delta z_{f,t+1}))$			0.924	0.914	0.2	-0.362
$Corr(m_d, m_f)$			0.965	0.972	0.68	0.731
$Std(M)/E(M)$			32.75	32.68	6.13	22.46
$corr(m_d^{SR}, m_f^{SR})$			0.178	0.184	0.41	0.731
$corr(m_d^{LR}, m_f^{LR})$			0.944	0.954	0.763	n.a.
$Std(m^{SR})$			1.52	1.518	1.664	22.4
$Std(m^{LR})$			31.55	31.47	4.495	n.a.

lated (0.93 vs 0.73) and exchange rates less volatile (7.60% vs 8.62%). In addition, in line with our model's risk-sharing mechanism induced by recursive preferences, faster adoption generates more common variation in the economy's future prospects, i.e. $corr(m_d^{LR}, m_f^{LR})$ increases. As a result, the difference between the cross-country correlation of consumption growth and that of stock market returns increases. In Section 6 we provide suggestive empirical evidence to support this mechanism.

Why endogenous growth?–the EXO case. When growth is exogenous, the risk propagation mechanism at the core of our baseline calibration is muted. Put simply: short-run risk, which results from the exogenous productivity process, does not have significant long-run effects on growth. In this case, the power of recursive preferences is limited. Even though recursive preferences make agents fear variations in future utility, such variation is too small to have significant quantitative effects on equilibrium asset prices. Indeed, the volatility of the long-run component of the SDF is only about 4.5%, or roughly 7 times smaller than what is obtained from the baseline model with endogenous growth. Similarly, the Sharpe Ratio declines to 0.06, and both the volatility of the exchange rate (4.88%) and the cross-country correlation in stock market returns (0.46) are lower than their empirical counterparts.

Why recursive preferences?–the CRRA case. Recursive preferences are crucial to our mechanism because they allow for realistic dynamics of asset prices without compromising the model's performance with respect to macroeconomic quantities. The last column of Table 2 reports the results obtained in the standard CRRA case. The dynamics of macroeconomic quantities within each country are only marginally changed, but the reported cross-country moments reveal some serious shortcomings: in particular, the CRRA case cannot account for the sizable wedge observed in the data between cross-country correlations in consumption growth and stock market returns. The cross-country correlation in consumption growth is too high (0.73), and the cross country correlation in the stock market excess returns is 0.96. Furthermore, the cross-country correlation of R&D expenditures is virtually zero because a positive productivity shock in one country has almost no impact on the prospects of future growth of the other country. Finally, the average of the risk-free rate is too high (19.98%)—a manifestation of the well-known risk-free rate puzzle.

5.3 Long-Run Risk Within and Across Countries

In this section, we investigate the mechanism at the heart of the model: the endogenous generation of long-run risk and its international diffusion.

Endogenous long-run risk. Growth within a country is risky as it is tied to discounted future profits of innovations. Iterating forward equation (10), the economy's growth rate can be expressed as the present discounted value of domestic and foreign profits as follows

$$\Delta N^d_{d,t+1} = (1-\phi) + E_t \left[\chi^{\frac{1}{\eta}} \sum_{j=1}^{\infty} M_{d,t+j} (1-\phi)^{j-1} (\pi^d_{d,t+j} + \pi^f_{d,t+j}) \right]^{\frac{\eta}{1-\eta}},$$

where $M_{d,t+j} \equiv \prod_s^j M_{d,t+s}$ is the j-period SDF.[10]

The novelty of our paper is as follows. In our international setting, intermediate producers can sell new varieties both to domestic final producers and, once they have been adopted, to foreign final producers. The profits from selling these varieties are discounted at the SDF from equation (3) and determine the characteristics of equilibrium growth in our model. With recursive preferences and a preference for early resolution of risk, agents fear variations on the economy's future prospects. The entire future distribution of risk matters. The more productivity and consumption growth are predictable, the more volatile the SDF. The result is a volatile endogenous growth path, which exhibits a quantitatively substantial long-run risk. In our baseline calibration expected productivity and consumption growth are highly persistent, with an autocorrelation coefficient of 0.915 and 0.913, respectively. The SDF is volatile, resulting in an annualized Share ratio of roughly one third. The success of our model in generating large long-run risk does not come at the expense of other moments. Indeed, our model matches quite well the implied dynamics of realized macroeconomic quantities, such as consumption and productivity growth, R&D expenditures and the number of varieties. Means, standard deviations, and cross-country correlations of these variables are in line with their empirical counterparts. This is due to the mechanism of international diffusion described below.

International diffusion of long-run risk. In our model, long-run risk diffuses internationally through trade in intermediate goods. When financial markets are complete, long-run risk

[10] Kung and Schmid (2015) derive a similar expression in the context of a closed economy.

must be highly correlated across countries. To see this, consider the risk-sharing condition in equation (21). In logs, $\Delta \log \mathcal{E}_{t+1} = m_{f,t+1} - m_{d,t+1}$. That is, the currency depreciation rate is equal to the difference between the foreign and domestic SDFs. In the data, the volatility of the currency depreciation is about 8%. Hansen and Jagannathan (1991) show that the SDF must have a volatility of, at least, 30-40%. Therefore, the correlation between the domestic and foreign SDFs must be close to one. Unfortunately, in the data, realized consumption growth is neither volatile enough nor highly correlated across countries. Therefore, the required high volatility and high correlation of the SDFs cannot come from its short-run components, but must come from the components capturing the continuation utility and the riskiness of future growth, i.e, $M^{LR}_{t+1} = \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma-\theta}$.

In our baseline calibration, the cross-country correlation of the SDFs is as high as 0.965 and is almost exclusively due to long-run risk. Indeed, the short-run component of the SDFs has a low cross-country correlation of 0.178, matching the cross-country correlation observed in the data for consumption growth. The sizable cross-country correlation in the long-run component of the SDFs manifests itself in a large cross-country correlation of expected consumption growth (0.765) and expected productivity growth (0.924). As a result, the equilibrium dynamics of the SDFs imply a volatility of the currency depreciation rate in our baseline calibration of 8.62%, which is consistent with the empirical value.

A bivariate VAR model of international long-run risk. The high cross-country correlation of expected consumption growth that we obtain in our baseline calibration is a consequence of the spillover effect we highlighted in our reduced form model of Section 2. We now conduct the same exercise using simulated data from our model. In particular, for each of the 5,000 simulations of 72 quarters from our baseline calibration, we estimate the following two-dimensional VAR(1) process

$$\begin{pmatrix} E_t(\Delta c^d_{t+1} - \mu) \\ E_t(\Delta c^f_{t+1} - \mu) \end{pmatrix} = \Phi \begin{pmatrix} E_{t-1}(\Delta c^d_t - \mu) \\ E_{t-1}(\Delta c^f_t - \mu) \end{pmatrix} + \Sigma \begin{pmatrix} \varepsilon^d_{t+1} \\ \varepsilon^f_{t+1} \end{pmatrix} ,$$

where μ is the unconditional mean of consumption growth, Φ is the autocorrelation matrix and Σ is the variance-covariance matrix associated with the innovation $(\varepsilon^d_{t+1}, \varepsilon^f_{t+1})$. Unlike the restricted example of Section 2, we let the model tell us the correlation structure in the innovations. We obtain the following results:

$$\Phi = \begin{pmatrix} 0.89 & 0.01 \\ 0.01 & 0.89 \end{pmatrix} \quad ; \qquad \Sigma = \begin{pmatrix} 0.165 & 0.118 \\ 0.118 & 0.165 \end{pmatrix} \quad .$$

Expected consumption growth is volatile and its international correlation structure depends not only on the cross-country correlation in the innovations but also on the spillover effect. Indeed, the off-diagonal elements of the autocorrelation matrix are positive and significantly different from zero. We can also estimate moments associated with this VAR specification. We find that expected consumption growth, our measure of long-run risk, is highly persistent *within* a country ($corr(E_t(\Delta c_{t+1}), E_{t-1}(\Delta c_t)) = 0.903$) and *across* countries ($corr(E_t(\Delta c^d_{t+1}), E_t(\Delta c^f_{t+1})) = 0.744$).

The role of the speed of adoption. We conclude this section with a sensitivity analysis of our results to changes in the speed of adoption, ϑ^f_d, the key parameter governing the international diffusion of long-run risk. We show that, in general, as the strength of the spillover mechanism increases, so does the correlation of international long-run risk, with a small caveat described below.

We conduct the following exercise. We consider the entire range of our empirical estimates of ϑ^f_d and solve and simulate the model for that range. Then, we focus on the effect of increasing the speed of adoption on the moments that are most affected by the international diffusion of long-run risk. Figure 1a shows how the cross-country correlation of expected consumption growth changes as we vary ϑ^f_d. We find that it is increasing and it ranges from 0.75, with a $\vartheta^f_d = 0.035$, to 0.78, with $\vartheta^f_d = 0.05$. That is, the faster the speed of adoption, the more long-run risk diffuses across countries.

Figure 1: Cross-country correlations of $E_t(\Delta c_{t+1})$ and $E_t(\Delta z_{t+1})$ as a function of the speed of adoption ϑ_d^f.

(a) Cross-country corr. of future cons. growth

(b) Cross-country corr. of future prod. growth

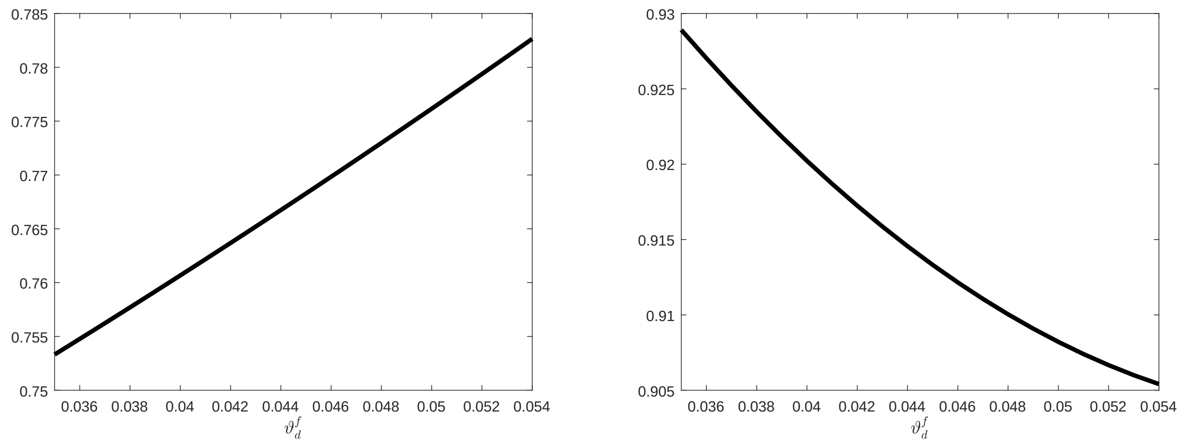


Figure 1b shows that, in our baseline calibration, the cross-country correlation in expected productivity growth decreases with ϑ_d^f. This result, however, depends crucially on ν, the parameter governing the elasticity of substitution between domestic and foreign goods. To see this, we derive the following expression for the expected growth of the endogenous component of productivity:

$$E_t \Delta \log(Z_{d,t+1}^{ENDO}) \approx \left[h\chi \left(\frac{S_{dt}}{N_{dt}^d} \right)^{\eta} + \left(\frac{h}{1-h} \right)^{\frac{\nu}{\nu-1}} \frac{\bar{N}_f^d}{\bar{N}_d^d} \left(\frac{\nu}{\nu-1} E_t \Delta \log(\mathcal{E}_{t+1}) + (1-\phi)\vartheta_d^f \chi \left(\frac{S_{ft}}{N_{ft}^f} \right)^{\eta} \frac{N_{ft}^f}{N_{ft}^d} \right) \right] \quad (23)$$

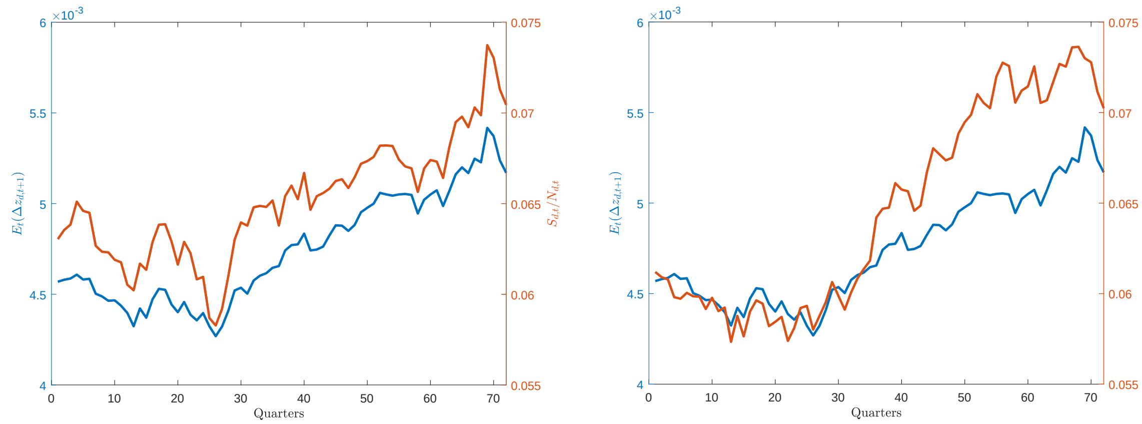
where $\bar{X}$ represents the steady-state value of variable X.[11]

Equation (23) shows that future expected productivity growth in each country depends both on its own R&D intensity and on the R&D intensity of the foreign country that gets diffused through adoption. In Figure 2 we look at this relation and plot simulated paths of expected domestic productivity growth together with domestic R&D intensity, S_d/N_d, and foreign R&D intensity, S_f/N_f. Consistently with equation (23), the two graphs reveal that expected domestic productivity growth tracks closely both domestic and foreign R&D intensity.

[11]Details of the derivation are described in Appendix C.

Figure 2: Expected productivity growth $E_t(\Delta z_{t+1})$ and domestic and foreign R&D intensity.

(a) Domestic R&D intensity (b) Foreign R&D intensity



We can also use equation (23) to understand how the correlation of future expected productivity growth between the domestic and the foreign economy depends on the interplay between the speed of adoption and the elasticity of substitution across goods. In particular, the cross-country correlation of expected future productivity growth depends on two factors: (i) the relative contribution of domestic and foreign R&D, and (ii) the effect of changes in the real exchange rate. As ϑ_d^f increases, the domestic final producer can adopt more foreign technologies. Hence, keeping the real exchange rate constant, foreign R&D intensity plays a relatively more important role relative to domestic R&D intensity. The same in true in the foreign economy. As a result, as ϑ_d^f goes up, one would expect an increase in the correlation of future expected productivity growth between the domestic and foreign economy. However, changes in the real exchange affect domestic and foreign expected productivity in opposite directions. To see this, consider the foreign counterpart of equation (23):

$$E_t \Delta \log(Z_{f,t+1}^{ENDO}) \approx \left[h\chi \left(\frac{S_{ft}}{N_{ft}^d}\right)^{\eta} + \left(\frac{h}{1-h}\right)^{\frac{\nu}{\nu-1}} \frac{\bar{N}_d^f}{\bar{N}_f^f} \left(-\frac{\nu}{\nu-1} E_t \Delta \log(\mathcal{E}_{t+1}) + (1-\phi)\vartheta_d^f \chi \left(\frac{S_{dt}}{N_{dt}^f}\right)^{\eta} \frac{N_{dt}^d}{N_{dt}^f} \right) \right] \quad (24)$$

Notice the *minus* sign in front of the exchange rate depreciation, so that $-\nu/(\nu-1)$ is positive. An increase in $\mathcal{E}$ drives *down* domestic future expected productivity growth in equation (23), but it *increases* the foreign future expected productivity growth in equation (24). The lower the elasticity of substitution, the stronger this effect. When the elasticity of substitution between domestic and foreign varieties is large enough, the exchange rate channel is relatively weak and the correlation of future expected productivity growth increases with ϑ_d^f. For lower elasticities of substitution, however, that is not the case, as the effect of the real exchange rate move in opposite directions in the domestic and foreign future expected productivity growth, unless we consider values of ϑ_d^f that are implausible high relative to our empirical estimates. Figure 3 shows that when we increase the elasticity of substitution across goods to 2 (*i.e.*, $\nu = 0.5$), the cross-country correlation of expected future productivity growth increases as we increase the speed of adoption. However, a larger eleasticity of substitution across goods reduces the equilibrium level of cross-country correlation of expected productivity. For the range of ϑ_d^f that we consider in our analysis, the cross-country correlation is always above 0.9 in our baseline calibration with $\nu = 0.4$, whereas for $\nu = 0.5$ it is always below 0.88 (*cfr.* Figure 1b with Figure 3).

Figure 3: Cross-country correlation of expected productivity growth $E_t(\Delta z_t)$ as a function of ϑ_d^f. The elasticity of substitution across goods is set equal to 2 ($\nu = 0.5$).

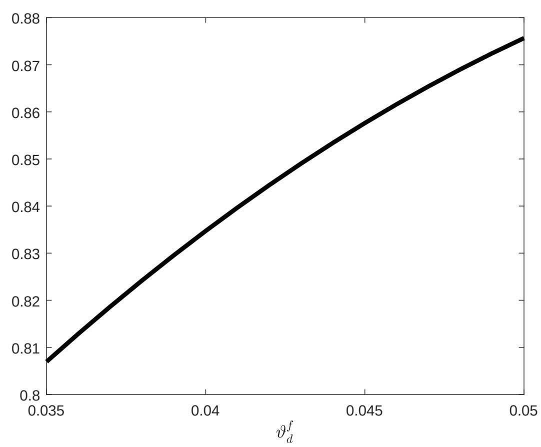


Figure 4 confirms that, with a stronger spillover effect, long run risk diffuses more across countries. Indeed, the SDFs becomes more correlated and, with complete markets, exchange rates become less volatile. For the range of our estimated speed of adoption, the standard deviation of the currency depreciation rate drops from 9.30% to 6.78%. Finally, we notice that as SDFs become more correlated, so do stock market returns. Figure 5 shows that as we increase ϑ_d^f, the correlation of stock market returns increases from 0.48 to 0.98.

Figure 4: Cross-country correlation of SDFs and volatility of currency depreciation rate.

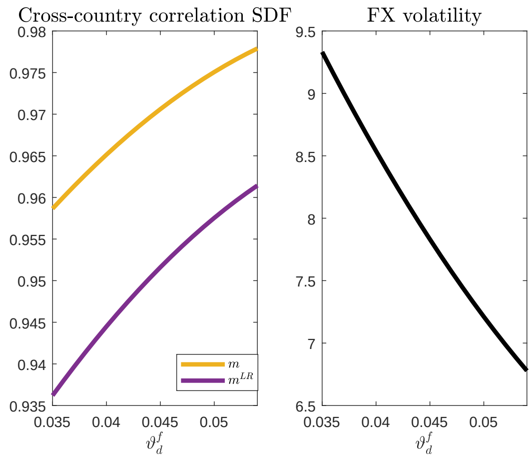
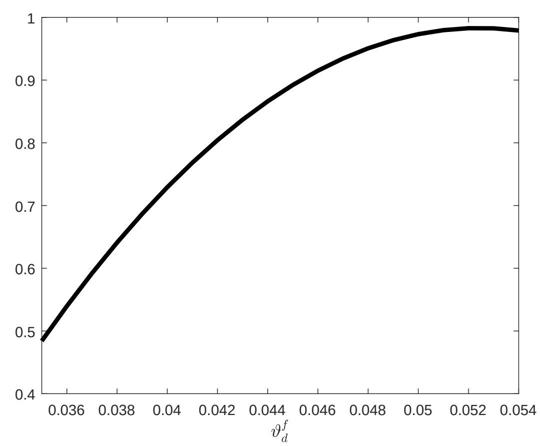


Figure 5: Cross-country correlation of stock market excess returns.



6 Innovation, Trade, and Asset Prices: Empirical Evidence

In this section we investigate the empirical relevance of our proposed mechanism: domestic R&D spending embodied in intermediate products increases both domestic and foreign aggregate productivity growth with a distinct effect on asset prices. Stock market returns become more correlated and exchange rates become less volatile. We first focus on a measure of R&D embodied in international trade. Then, we consider a measure that closely tracks the speed of adoption in our model, ϑ_d^f.

There is a large empirical literature studying the role that R&D embodied in intermediate goods has on the productivity of a country (Keller (1998), Coe and Helpman (1995), Nishioka and Ripoll (2012)). The main idea behind these studies is that technology transfers across countries contribute to increases in productivity of the country receiving such technologies. Technology can transfer across countries in various ways: though international trade, multinational production, human capital accumulation. Several studies have found that technology embodied in intermediate goods that are traded is an important channel for international technology diffusion. On the theory side, Romer (1990) and Grossman and Helpman (1991) have developed models in which both domestic and foreign intermediate goods increases productivity. That is the mechanism at work in our model. Because trade is Armington, final producers want to use as many intermediate goods as possible. On the empirical side, research has found a tight link between R&D embodied in trade and total factor productivity. The mechanism proposed in our paper is based on these studies.

In this section, we provide suggestive empirical evidence of the main mechanism of our model. We are not claiming this is the only channel through which R&D is shared across countries, but our findings suggest the existence of a relationship between asset pricing moments and technology embodied in intermediate goods.

Our main data sources are as follows: for international trade, the UN COMTRADE; for R&D, the OECD Science and Technology Indicators (Business Enterprise R&D (BERD) as a % of GDP); and for asset prices, Global Financial Data and Ken French's website.[12]. Our main data set covers the 1996–2013 period for a sample of 19 countries.[13] The choice of countries and time period was determined based on the availability of data for asset pricing, trade, and R&D. Details on these sources–and on the construction of measures used in our analysis–are given in Appendix D.

We analyze the correlations between broad measures of asset prices and measures of international trade and R&D. The goal is to determine whether the mechanism that we propose (i.e., R&D embodied in trade) plays any role in asset prices. We consider two statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the exchange rate. Our model predicts that pairs of countries sharing more R&D have more

[12]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[13]The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States. For exchange rates, we use the same period but eliminate all Eurozone countries (save Germany) and Denmark, which has instituted a managed exchange rate versus the Euro.

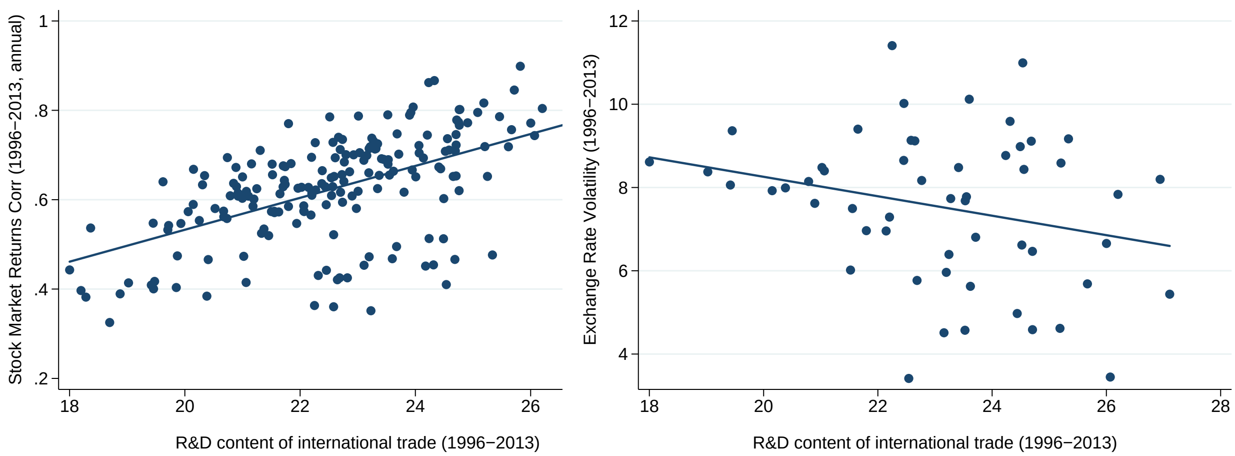


Figure 6: R&D content of trade and asset prices: time-series average for each country pair.

correlated stock market returns and less volatile exchange rates.

We start with monthly observations and construct annual, non-overlapping measures of cross-country correlation—for stock market returns—and volatility—for the exchange rate—for each ordered country-pair.[14] We then use data on R&D and bilateral trade flows to construct a measure of the R&D content of trade. This is the key measure reflecting our mechanism of how R&D is shared across countries though international trade. For this purpose, we start by constructing the R&D stock for each country in our sample. Given a country's R&D intensity—defined as R&D expenditures over gross domestic product (GDP)—we apply the so-called perpetual inventory method using an annual depreciation rate of 15% (as in Coe and Helpman 1995; Nishioka and Ripoll 2012). Next, we follow the empirical literature and obtain a measure of the R&D content of international trade for a particular pair of countries i and j ($X_{i,j}^{\text{R\&D}}$), as

$$X_{i,j}^{\text{R\&D}} = \frac{X_{i,j}}{GDP_j} R\&D_j + \frac{X_{j,i}}{GDP_i} R\&D_i \tag{25}$$

where $X_{i,j}$ is total trade between exporter j and importer i, $R\&D_i$ is the stock of R&D in country i, and GDP_i is the gross domestic product of country i. This measure is constructed for each year in our sample.

We then asses the relationship between our measures of asset prices and the R&D content of international trade. Figure 6 reveals that pairs of countries whose trade is characterized

[14]We also consider 60-month (5-year) overlapping measures with a 12-month overlap, and also the average across all periods.

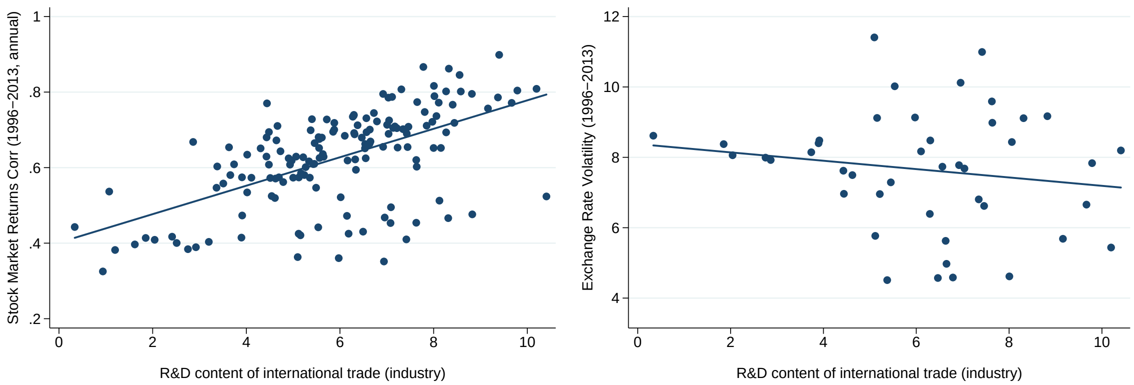


Figure 7: R&D content of trade and asset prices (industry measure): time-series average for each country pair.

by greater R&D content exhibit more highly correlated stock market returns and less volatile exchange rates. In the Figure's two graphs, each solid circle represents the time-series average over the 1996-2013 period of each country pair in our sample. We have 171 unique country pairs with regard to the stock market correlation (left panel) and 108 unique country pairs with regard to the exchange rate volatility (right panel).

A large part of the empirical literature on productivity and trade has used aggregate data at the country level for R&D and bilateral trade flows for a cross-section analysis of the impact of the R&D content of international trade on aggregate variables. More recently, Nishioka and Ripoll (2012) use more disaggregated trade and R&D data. Following their work closely, we conduct a robustness analysis in which a measure of R&D embodied in intermediate inputs is calculated at the industry level. We use their global input-output matrix of 13 manufacturing industries and 32 countries for the years 1995, 2000, and 2005. Our more aggregated measure has a longer time series but fails to account for the disaggregation of R&D expenditures at the industry level. We then evaluate the extent to which this new measure correlates with our moments of asset prices. As Figure 7 shows, the results are broadly in line with those obtained using more aggregated measures. Country pairs that share more R&D through trade, using the input-output structure of the data, have more correlated stock market returns and less volatile exchange rates. Taken together, these graphs provide suggestive evidence that R&D and international trade can inform us on the international dynamics of asset prices.

Finally, we explore in more detail the relation between the speed of adoption, international trade and asset prices. To do that, we compute a value for the speed of adoption between each country-pair, using a similar approach to the one used in our calibration. Specifically, for each country-pair, we regress the log of the extensive margin of trade against the average stock of R&D expenditures among the two countries including year fixed effects.[15]

We then analyze the correlation between the estimated speed of adoption for each country-pair with: (i) international trade; and (ii) asset prices. Figure 8 shows that country-pairs with faster speed of adoption have a higher trade intensity and a higher extensive margin of trade.

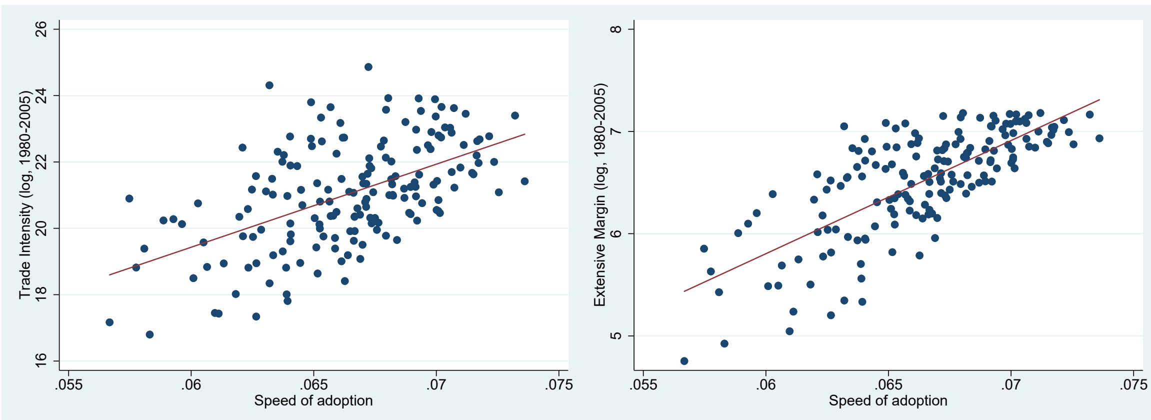


Figure 8: International trade and the speed of adoption.

Consistently with our suggestive evidence presented previously, we find in Figure 9 that country-pairs with a faster speed of adoption also have a larger cross-country correlation of stock market returns and a lower volatility of the exchange rate.

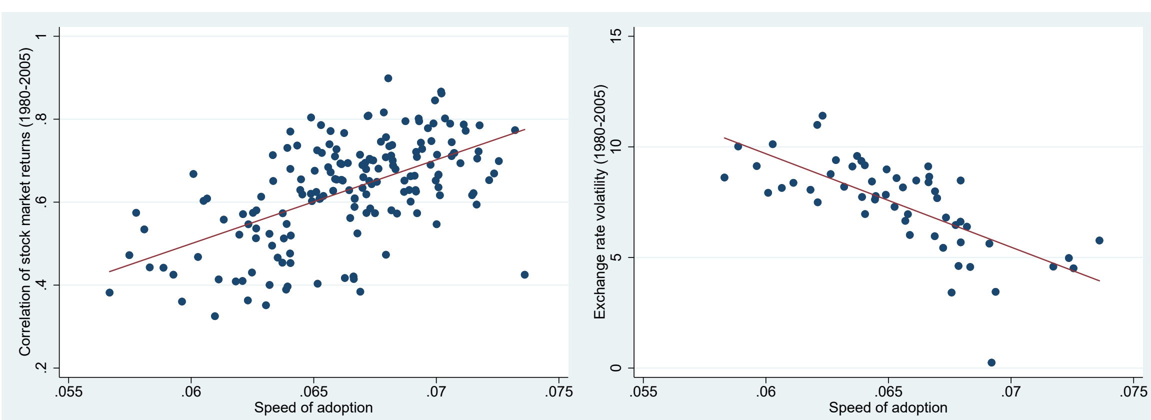


Figure 9: Asset prices and the speed of adoption.

[15]This is slightly different from what we do in our calibration section. There, we use the entire panel of data to compute one ϑ_d^f that corresponds to the average country. Here, we are calculating one ϑ_d^f for each-country pair.

Ideally, we would like to use data on the R&D content of each traded variety. Unfortunately, such data is not available for a large sample of countries. However, we believe that the empirical analysis of this section suggests that the mechanism of the international diffusion of R&D proposed by our model may be informative about the joint dynamics of international macro quantities and asset prices.

7 Conclusion

In this paper, we develop a multicountry general equilibrium model in which long-run risk arises endogenously in each country and then diffuses internationally. Specifically, we conduct a quantitative analysis of a symmetric, two-country, endogenous growth model of innovation and international adoption of foreign varieties. We have shown—both theoretically and by way of a calibration exercise—that the joint presence of endogenous growth and recursive preferences has a significant impact on the equilibrium dynamics of both quantities and asset prices. In particular, we have seen that when both ingredients are present, the model generates expected consumption growth processes that are volatile, highly autocorrelated within the country and highly correlates across countries. This is consistent with what has been advocated by the exogenous long-run risk literature in order to rationalize several international macroeconomic and asset-pricing puzzles. Specifically, our model generates a low and smooth risk-free rate, large Sharpe ratios, and exchange rate volatility and cross-country correlation of macroeconomic quantities that are consistent with the data.

We provide suggestive empirical evidence of our proposed mechanism. In particular, we show that country pairs with a higher R&D content of international trade have more highly correlated stock market returns and less volatile exchange rates.

The model developed in this paper could be extended to address other international asset-pricing puzzles. For instance, relaxing the symmetry assumption would allow one to analyze whether our mechanism might help explain observed deviations in the uncovered interest parity condition and the profitability of the currency carry trade. We leave these and other issues for future research.

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Appendix

A Model Equations

This appendix consists of all the equations used to characterize the domestic economy. The foreign economy is represented by a set of analogous equations.

Preferences

$$U_{d,t} = \left\{(1-\beta)C_{d,t}^{\theta} + \beta\left(E_t\left(U_{d,t+1}^{1-\gamma}\right)\right)^{\frac{\theta}{1-\gamma}}\right\}^{\frac{1}{\theta}}$$

Stochastic discount factor

$$M_{d,t+1} = \beta\left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1}\left(\frac{U_{d,t+1}}{E_t\left(U_{d,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma-\theta}$$

Final producers

$$Y_{d,t} = (Z_{d,t}L_{d,t})^{1-\alpha}K_{d,t}^{\alpha}$$

Labor

$$L_{d,t} = 1$$

Aggregate productivity

$$Z_{d,t} \equiv \Omega_{d,t}\left(\tilde{A}\right)^{\frac{1}{1-\alpha}}\left[hN_{d,t}^{d} + \left(\frac{h}{1-h}\mathcal{E}_t\right)^{\frac{\nu}{\nu-1}}N_{f,t}^{d}\right]$$

$$\tilde{A} = (\xi\nu)^{\frac{\xi}{1-\xi}}$$

$$\Omega_{d,t} = e^{a_{d,t}}$$

$$a_{d,t} = \varphi \ a_{d,t-1} + \varepsilon_{d,t}$$

First order condition of labor

$$W_{d,t} = (1-\alpha)(1-\xi)\frac{Y_{d,t}}{L_{d,t}}$$

First order condition of investment

$$q_{d,t} = \frac{1}{\Lambda'_{d,t}}$$

$$1 = E_t\left[M_{d,t+1}\left\{\frac{1}{q_{d,t}}\left(\alpha(1-\xi)\frac{Y_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}(1-\delta) - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\Lambda_{d,t+1}\right)\right\}\right]$$

Law of motion of capital

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda_{d,t}K_{d,t}$$

Investment adjustment costs

$$\Lambda_{d,t} \equiv \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right) = \frac{\alpha_1}{\zeta}\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta} + \alpha_2$$

$$\Lambda'_{d,t} = \alpha_1\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta-1}$$

Demand for domestic intermediate goods

$$X^d_{d,t} = (h\xi\nu Y_{d,t}G^{-\nu}_{d,t})^{\frac{1}{1-\nu}}$$

Demand for foreign intermediate goods (imports)

$$X^d_{f,t} = (\mathcal{E}_t^{-1}\nu Y_{d,t}G^{-\nu}_{d,t}\xi(1-h))^{\frac{1}{1-\nu}} = X^d_{d,i,t}\left(\mathcal{E}_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}$$

Materials (intermediate goods)

$$G_{d,t} = \xi\nu Y_{d,t}\left[hN^d_{d,t} + \left(\frac{h}{1-h}\mathcal{E}_t\right)^{\frac{\nu}{\nu-1}} N^d_{f,t}\right]^{\frac{1-\nu}{\nu}}$$

Profits of intermediate producers

$$\Pi_{d,t}N_{d,t}^{d} = \pi_{d,t}^{d}N_{d,t}^{d} + \pi_{d,t}^{f}N_{d,t}^{f}$$

Profits of domestic producers in the domestic market

$$\pi_{d,t}^{d} = \left(\frac{1}{\nu} - 1\right)X_{d,t}^{d}$$

Profits of domestic producers in the foreign market

$$\pi_{d,t}^{f} = \left(\frac{1}{\nu} - 1\right)X_{d,t}^{f}$$

Present Discounted Value (PDV) of a domestic producers selling in the domestic market

$$V_{d,t}^{d} = \pi_{d,t}^{d} + (1-\phi)E_t[M_{d,t+1}V_{d,t+1}^{d}]$$

PDV of a domestic producers selling in the domestic market

$$V_{d,t}^{f} = \pi_{d,t}^{f} + (1-\phi)E_t[M_{d,t+1}V_{d,t+1}^{f}]$$

PDV of a domestic producers not-yet selling in the domestic market

$$J_{d,t}^{f} = (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta_d^f\pi_{dt+1}^{f} + (1-\vartheta_d^f)J_{d,t+1}^{f}\right)\right]$$

Value of an innovation

$$V_{d,t} = V_{d,t}^{d} + J_{d,t}^{f}$$

Law of motion of new technologies

$$N_{d,t+1}^{d} = \vartheta_{d,t}S_{d,t} + (1-\phi)N_{d,t}^{d}$$

$$\vartheta_{d,t} = \frac{\chi(N_{d,t}^{d} + \iota N_{f,t}^{d})}{S_{d,t}^{1-\eta}(N_{d,t}^{d})^{\eta}}$$

Free entry condition of innovation

$$S_{dt} = E_t\left[M_{d,t+1}V_{d,t+1}\right]\left(N_{d,t+1}^{d} - (1-\phi)N_{d,t}^{d}\right)$$

Law of motion of adopted technologies

$$N^f_{d,t+1} = \vartheta^f_d(1-\phi)(N^d_{d,t} - N^f_{d,t}) + (1-\phi)N^f_{d,t}$$

Resource Constraint

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N^d_{d,t}X^d_{d,t} + N^f_{d,t}X^f_{dt}$$

International risk sharing

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{M_{f,t+1}}{M_{d,t+1}}$$

B Deriving the Stock Market

Dividends are generated by the final producers, the intermediate producers, and the innovators. The stock market is the present discounted value (PDV) of the future dividends generated by all the firms in the economy. Optimality implies the following asset-pricing condition:

$$\mathcal{P}_{d,t} \quad = \quad E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})] \quad ,$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price, and $\mathcal{D}_{d,t}$ is the aggregate market dividend. Substituting forward, we have

$$\mathcal{P}_{d,t} = E_t \sum_{i=0}^{\infty} M_{d,t+i+1}\mathcal{D}_{d,t+i+1}$$

Total dividends are

$$\mathcal{D}_{d,t} = D_{d,t} + N^d_{d,t}\pi^d_{d,t} + N^d_{f,t}\pi^d_{f,t} - S_{d,t} \quad ,$$

with the dividends of the final producers, $D_{d,t}$, evolving according to

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - N^d_{d,t}P^d_{d,t}X^d_{d,t} - N^d_{f,t}P^d_{f,t}X^d_{f,t} \quad .$$

Consider the PDV of the dividends of the final producers, $\mathcal{P}^{tan}_{d,t}$. We have

$$\mathcal{P}^{tan}_{d,t} = E_t\left[\sum_{i=0}^{\infty} M_{d,t+i+1}D_{d,t+i+1}\right]$$

or, in recursive form, $\mathcal{P}^{tan}_{d,t} = E_t\left[M_{d,t+1}\left(\mathcal{P}^{tan}_{d,t+1} + D_{d,t+1}\right)\right]$.

Result. $\mathcal{P}^{tan}_{d,t} = q_{d,t}K_{d,t+1}$.

Proof. Consider the following expression:

$$E_t(M_{d,t+1}D_{d,t+1}) = E_t\left[M_{d,t+1}\left(Y_{d,t+1} - I_{d,t+1} - W_{d,t+1}L_{d,t+1} - N^d_{d,t+1}P^d_{d.t+1}X^d_{d,t+1} - N^d_{f,t+1}P^d_{f,t+1}X^d_{f,t+1}\right)\right] \quad .$$

From the first-order condition (FOC) of labor it follows that

$$W_{d,t+1}L_{d,t+1} = (1-\alpha)(1-\varepsilon)Y_{d,t+1} \quad .$$

We can use the FOC for intermediate producers to rewrite the expression

$$N^d_{d,t}P^d_{dt}X^d_{d,t} + N^d_{f,t}P^d_{f,t}X^d_{f,t}$$

or, substituting for the prices of intermediate goods,

$$N^d_{d,t}\frac{1}{\nu}X^d_{d,t} + N^d_{f,t}\frac{1}{\nu}\mathcal{E}_t X^d_{f,t} \quad .$$

Now using

$$X^d_{f,t} = X^d_{d,t}\left(\mathcal{E}_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}},$$

we obtain

$$\left(N^d_{d,t}\frac{1}{\nu} + N^d_{f,t}\frac{1}{\nu}\mathcal{E}_t\left(\mathcal{E}_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t}(\mathcal{E}_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)\frac{1}{\nu}X^d_{d,t} \quad .$$

Similarly, using

$$X^d_{d,t} = \left(h\xi\nu Y_{d,t}G^{-\nu}_{d,t}\right)^{\frac{1}{1-\nu}}$$

and substituting $G_{d,t} = \xi\nu Y_{d,t}\left[hN^d_{d,t} + \left(\frac{h}{1-h}\mathcal{E}_t\right)^{\frac{\nu}{\nu-1}}N^d_{f,t}\right]^{\frac{1-\nu}{\nu}}$, we have

$$X^d_{d,t} = \xi\nu Y_{d,t}\left[hN^d_{d,t} + \left(\frac{h}{1-h}\mathcal{E}_t\right)^{\frac{\nu}{\nu-1}}N^d_{f,t}\right]^{-1} \quad .$$

Plugging this expression into the total spending for intermediate producers, we have

$$\left(N^d_{d,t}\frac{1}{\nu} + N^d_{f,t}\frac{1}{\nu}\mathcal{E}_t\left(\mathcal{E}_t\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t}(\mathcal{E}_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)\frac{1}{\nu}X^d_{d,t} =$$

$$= \left(N^d_{d,t} + N^d_{f,t}(\mathcal{E}_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)\frac{1}{\nu}\varepsilon\nu Y_{d,t}\left(N^d_{d,t} + N^d_{f,t}(\mathcal{E}_t)^{\frac{\nu}{\nu-1}}\left(\frac{h}{1-h}\right)^{\frac{1}{\nu-1}}\right)^{-1} = \varepsilon Y_{d,t} \quad .$$

Finally, consider the FOC for investment and rearrange it to obtain

$$q_{d,t}K_{d,t+1} = E_t\left[M_{d,t+1}\left(\alpha(1-\varepsilon)Y_{d,t+1} - I_{d,t+1}\right)\right] + E_t\left[M_{d,t+1}q_{d,t+1}\left((1-\delta)+\Lambda_{d,t+1}\right)K_{d,t+1}\right] \quad .$$

From the law of motion for capital, it follows that

$$\left(\frac{(1-\delta)+\Lambda_{d,t+1}}{K_{d,t+1}}\right) = \frac{K_{d,t+2}}{K_{d,t+1}}$$

substituting into the previous expression, then yields

$$q_{d,t}K_{d,t+1} = E_t\left[M_{d,t+1}\left(\alpha(1-\varepsilon)Y_{d,t+1} - I_{d,t+1}\right)\right] + E_t\left[M_{d,t+1}q_{d,t+1}K_{d,t+2}\right] \quad .$$

Letting $\hat{q}_t = \mathcal{E}_t K_{t+1}$, solving the expression just displayed recursively, and imposing the standard transversality condition, we have

$$\hat{q}_t = E_t \left[\sum_{i=0}^{\infty} M_{t+i+1} \left(\alpha(1-\varepsilon)\, Y_{t+i+1} - I_{t+i+1} \right) \right] \quad .$$

Combining all these results, we finally obtain

$$\begin{aligned} \mathcal{P}_{d,t}^{tan} &= E_t \left[M_{d,t+1} \left(\mathcal{P}_{d,t+1}^{tan} + D_{d,t+1} \right) \right] \\ &= E_t \left[\sum_{i=0}^{\infty} M_{d,t+i+1} \left(-I_{d,t+i+1} + (\alpha)(1-\varepsilon) Y_{d,t+i+1} \right) \right] = \hat{q}_{d,t} \end{aligned}$$

□

We now calculate the present discounted values of the remaining terms in the expression for the market dividends. When computing the PDV of the profits of all the existing intermediate producers, we need to account for the existence of two types of intermediate producers in the economy: firms currently selling to both the domestic and the foreign market; and firms that are not yet selling to the foreign market but that might do so in the future.

We start by calculating the PDV of each firm that currently sells to both the domestic and foreign market today. These firms continue selling to both markets until they disappear disappear with probability ϕ. Let's denote the present discounted value of the dividends of these firms as $\pi_{d,t}$, which are given in recursive form by

$$\pi_{d,t} = \pi_{d,t}^d + \pi_{d,t}^f + (1-\phi) E_t \left[M_{d,t+1} \pi_{d,t+1} \right] = V_{d,t}^d + V_{d,t}^f$$

where the last equality uses the definition of the value function for one firm selling only to the domestic market and the value function of one firm that is already selling to the foreign market. From the previous expression, the expected PDV of the future dividends for one firm that sells both in the domestic and in the foreign market, which is the component that we need to compute the stock market and we call V_{1t}, is

$$V_{1t} = (1-\phi) E_t \left[M_{d,t+1} \pi_{d,t+1} \right] = \left(V_{d,t}^d - \pi_{d,t}^d \right) + \left(V_{d,t}^f - \pi_{d,t}^f \right)$$

Finally, the PDV of the dividends of firms that only sell today in the domestic market but have a chance to sell tomorrow to the export market is given, in recursive form, by

$$\pi_{d,t}^d + (1-\phi) E_t \left[M_{d,t+1} \left(\pi_{d,t+1}^d + J_{d,t+1}^f \right) \right] = V_{d,t}^d + (1-\phi) E_t \left[M_{d,t+1} J_{d,t+1}^f \right]$$

From the previous expression, the expected PDV of the future dividends for a firm that sells only in the domestic market, which is the component that we need to compute the stock market and we call V_{2t}, is

$$V_{2t} = (1-\phi) E_t \left[M_{d,t+1} \left(\pi_{d,t+1}^d + J_{d,t+1}^f \right) \right] = \left(v_{d,t}^d - \pi_{d,t}^d \right) + (1-\phi) E_t \left[M_{d,t+1} J_{d,t+1}^f \right]$$

Since there are $N_{d,t}^f$firms selling to both the domestic and the foreign market and $\left(N_{d,t}^d - N_{d,t}^f \right)$ firms selling only to the foreign market, we can aggregate the previous expressions to obtain the component of the stock market that is driven by the already established intermediate producers as

$$N^f_{d,t} V_{1t} + (N^d_{d,t} - N^f_{d,t}) V_{2t} =$$

$$= N^f_{d,t}\left[\left(V^d_{d,t} - \pi^d_{d,t}\right) + \left(V^f_{d,t} - \pi^f_{d,t}\right)\right] + (N^d_{d,t} - N^f_{d,t})\left[\left(V^d_{d,t} - \pi^d_{d,t}\right) + (1-\phi)E_t\left[M_{d,t+1} J^f_{d,t+1}\right]\right]$$

$$= N^d_{d,t}\left(V^d_{d,t} - \pi^d_{d,t}\right) + N^f_{d,t}\left(V^f_{d,t} - \pi^f_{d,t}\right) + (N^d_{d,t} - N^f_{d,t})\left((1-\phi)E_t\left[M_{d,t+1} J^f_{d,t+1}\right]\right)$$

Therefore, there are three components to the stock market

1. Price of installed capital

$$q_{d,t} K_{d,t+1}$$

2. Value of intangible capital

$$N^d_{d,t}\left(V^d_{d,t} - \pi^d_{d,t}\right) + N^f_{d,t}\left(V^f_{d,t} - \pi^f_{d,t}\right)$$

3. Value of intangible capital that could possibly be sold abroad

$$(1-\phi)(N^d_{d,t+1} - N^f_{d,t+1})E_t\left[M_{d,t+1} J^f_{d,t+1}\right]$$

C Future productivity growth

Computing the expected future growth rate of productivity growth From equation (19),

$$Z^{ENDO}_{d,t} \propto \left[h N^d_{d,t} + \left(\frac{h}{1-h}\mathcal{E}_t\right)^{\frac{\nu}{\nu-1}} N^d_{f,t}\right], \tag{26}$$

Log-linearizing the expression around the steady state, taking growth rates and expectations, we have

$$\frac{\bar{Z}_d}{\bar{N}^d_d} E_t \Delta \log(Z^{ENDO}_{d,t+1}) \propto \left[h E_t \Delta \log(n^d_{d,t+1}) + \left(\frac{h}{1-h}\right)^{\frac{\nu}{\nu-1}} \frac{\bar{N}^d_f}{\bar{N}^d_d}\left(\frac{\nu}{\nu-1} E_t \Delta \log(\mathcal{E}_{t+1}) + E_t \Delta \log(n^d_{f,t+1})\right)\right], \tag{27}$$

where $\bar{X}$ reflect the steady-state value of variable X.

Now we need to obtain $E_t \Delta \log(n^d_{d,t+1})$ and $E_t \Delta \log(n^d_{f,t+1})$. From equation (10),

$$E_t \Delta \log(n^d_{d,t+1}) \approx (1-\phi) + \chi\left(\frac{S_{dt}}{N^d_{dt}}\right)^{\eta}$$

and from equation (12),

$$E_t \Delta \log(n^d_{f,t+1}) \approx (1-\phi)(1-\vartheta) + (1-\phi)\vartheta E_t \Delta \log(n^f_{f,t+1}) \frac{N^f_{ft}}{N^d_{ft}}$$

which can also be written as

$$E_t \Delta \log(n^d_{f,t+1}) \approx (1-\phi)(1-\vartheta^f_d) + (1-\phi)\vartheta^f_d \chi \left(\frac{S_{ft}}{N^f_{ft}}\right)^\eta \frac{N^f_{ft}}{N^d_{ft}}$$

Substituting into equation (27)

$$E_t \Delta \log(Z^{ENDO}_{d,t+1}) \approx \left[h\chi \left(\frac{S_{dt}}{N^d_{dt}}\right)^\eta + \left(\frac{h}{1-h}\right)^{\frac{\nu}{\nu-1}} \frac{\bar{N}^d_f}{\bar{N}^d_d} \left(\frac{\nu}{\nu-1} E_t \Delta \log(\mathcal{E}_{t+1}) + (1-\phi)\vartheta^f_d \chi \left(\frac{S_{ft}}{N^f_{ft}}\right)^\eta \frac{N^f_{ft}}{N^d_{ft}} \right) \right], \quad (28)$$

D Trade Data, Asset Prices, and Comovements

In this appendix, we describe the relevant trade data and asset prices, after which we construct the measures used in our analysis.

Trade Data

The source of our trade data is UN COMTRADE. We collect product data at the 6-digit level of disaggregation; these data are annual and cover the period 1996-2013. Our focus is on the trade that occurs between importer i (identified by its IISCODE) and exporter j (identified by its EISCODE). We therefore collect data on the product types being traded (using their 6-digits identifiers it) and the dollar value of the trade in each product (i.e., the per-product trade value).

To derive our preliminary statistics, we calculate the fraction of world trade and world GDP accounted for by the countries in our sample (those countries are listed in footnote 5).

From this data, we construct the following measures:

- Trade Intensity (i, j): $TI_{i,j}$, the sum of the trade value of all traded products
- Extensive margin (i, j): $EM_{i,j}$, the number (the"count") of different types of goods imported by country i from country j
- Intensive Margin (i, j): $IM_{i,j}$, "how much", in US dollars, country i is trading on average for each product imported from country j

In order to compare these numbers across pairs of countries, we normalize them while accounting for each country's GDP. These normalized measures are defined as

$$\tilde{TI}_{i,j} = \frac{TI_{i,j} + TI_{j,i}}{GDP_i + GDP_j}$$

Each country pair is ordered: i is the importer and j is the exporter, i.e., $TI_{i,j}$ will usually be different from $TI_{j,i}$

We want to make sure that the relationship $\tilde{TI}_{i,j} = \tilde{EM}_{i,j}\tilde{IM}_{i,j}$ holds, so that, taking logs, we can easily run linear regressions.

Asset Prices

We consider two main statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the currency depreciation rate. We start from monthly observations of stock market returns (from Ken French's website) and exchange rates (from Global Financial

Data). Then, using twelve monthly observation for each year of our sample, we construct the following measures:

Stock Market

- Annual cross-country stock market return correlations between country i and country j, $corr(r^s_{i,t}, r^s_{j,t})$, for each year t.

Exchange Rate

- Annual volatility of currency i depreciation rate with respect to currency j, $vol(\Delta q^j_{i,t})$, for each year t. The log depreciation rate for currency i with respect to currency j is defined as $\Delta q^j_{i,t} = q^j_{i,t} - q^j_{i,t-1}$, where $q^j_{i,t}$ is the log exchange rate level at time t for country i (in units of currency i per one unit currency j.