Capital Accumulation and Dynamic Gains from Trade *

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Abstract

We compute welfare gains from trade in a dynamic, multicountry model with capital accumulation and trade imbalances. We develop a gradient-free method to compute the exact transition paths for 44 countries following a trade liberalization. We find that (i) the gains are negatively correlated with size, measured by total real GDP, (ii) larger countries accumulate a current account surplus and financial resources flow from larger countries to smaller countries boosting consumption in the latter, (iii) countries with larger short-run trade deficits accumulate capital faster, (iv) the gains are nonlinear in the reduction in trade costs, and (v) capital accumulation accounts for substantial gains. The net foreign asset position before the liberalization and the intensities of tradables in investment goods production and consumption goods production are quantitatively important for the gains.

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1 Introduction

How large are the welfare gains from trade? This is an old and important question. This question has typically been answered in static settings by computing the change in real income from an observed equilibrium to a counterfactual equilibrium. In such computations, the factors of production and technology in each country are held fixed and the change in real income is immediate and is entirely due to the change in each country's trade share that responds to a change in trade frictions. Recent examples include Arkolakis, Costinot, and Rodríguez-Clare (2012) (ACR hereafter), who compute the welfare cost of autarky, and Waugh and Ravikumar (2016), who compute the welfare gains from frictionless trade.

We calculate welfare gains from trade in a dynamic multicountry Ricardian model where international trade affects the capital stock in each country in each period. Our environment is a version of Eaton and Kortum (2002) embedded in a two-sector neoclassical growth model, similar to Alvarez and Lucas (2017). There is a continuum of tradable intermediate goods that are used in the production of investment goods, final consumption goods, and intermediate goods. Each country is endowed with an initial stock of capital and investment goods augment the stock of capital. We add two features that affect the gains: (i) Endogenous trade imbalances and (ii) cross-country heterogeneity in the intensity of tradables in investment goods and in consumption goods. We endogenize the trade imbalances via asset trade across countries. This feature helps each country smooth its consumption over time. The second feature affects the response of the relative price of investment in each country after a trade liberalization and, hence, the rate of capital accumulation.

We calibrate the intensities of tradables using the World Input Output Database. We calibrate productivities and trade costs so that the steady state of the model reproduces the observed bilateral trade flows across 44 countries and the trade imbalances in each country. We then conduct a counterfactual exercise in which there is an unanticipated, uniform, and permanent 20 percent reduction in trade frictions in all countries. We compute the exact levels of endogenous variables along the transition path from the calibrated steady state to the counterfactual steady state and calculate the welfare gains using a consumption-equivalent measure as in Lucas (1987). Welfare gains from the trade liberalization accrue gradually in our model since capital accumulation is not immediate and our measure of gains includes the gradual transition from the initial steady state to the counterfactual steady state.

We find that (i) the gains are negatively correlated with size, measured by total real GDP—the gain for the U.S. is 4.4 percent but the gain for Bulgaria is 21 percent; (ii) the current account balance immediately after the liberalization is positively correlated with size—larger countries accumulate a current account surplus and financial resources flow from larger countries to smaller countries boosting consumption in the latter; (iii) half-life for capital accumulation is negatively correlated with short-run trade deficits—countries with larger short-run trade deficits accumulate capital faster; (iv) gains from trade are nonlinear—elasticity of gains with respect to reductions in trade costs is higher for larger reductions; (v) dynamic gains are 35 percent more that gains in a static model where capital is fixed and tradables are used only in the production of final goods and intermediate goods; and (vi) steady-state gains in a balanced-trade version of our dynamic model are 80 percent more than the static gains.

Trade liberalization affects gains in our model through two channels: Total Factor Productivity (TFP) and capital-labor ratio. The TFP channel is a familiar one in trade models. Trade liberalization results in a decline in home trade share and, hence, an increase in TFP, which increases output. This channel affects the entire path of consumption in a dynamic model. Trade liberalization also increases the rate of capital accumulation due to the decrease in the relative price of investment and the increase in TFP. In our model, investment goods production is more tradables-intensive than consumption goods production, so when price of tradables declines the relative price of investment declines. A lower relative price of investment alters the rate of transformation between consumption and investment, and helps allocate a larger share of output to investment without sacrificing consumption. This increases the investment rate and the capital-labor ratio, thereby increasing output and consumption along the transition path. The increase in capital-labor ratio is gradual as in the neoclassical growth model. The increase in TFP also affects the rate of capital accumulation since higher TFP implies a higher return to capital. In a static model, the capital-labor ratio channel is clearly absent.

The role of trade imbalances in generating the gains depends on net foreign asset positions before the liberalization and on the nature of the liberalization. Starting from a net foreign asset position of zero and balanced trade, gains from an unanticipated liberalization are not quantitatively affected by asset trades across countries. That is, after the liberalization, allowing for endogenous trade imbalances or restricting allocations to balanced trade does not affect the gains quantitatively. However, if the liberalization is anticipated then allowing for endogenous trade imbalances implies more gains relative to a world where each country's trade is balanced every period. In a world with asset trades across countries, the initial cross-country heterogeneity in net foreign asset positions also has important quantitative implications for gains from liberalization. An unanticipated trade liberalization increases the world interest rate on impact, which implies that countries with initial debt suffer and countries with initial positive assets benefit. That is, countries that start with a negative net foreign asset position lose relative to being in an environment where each country's initial net foreign asset position is zero.

The intensity of tradables also plays a quantitative role in the gains. Tradables are more intensive in investment goods production than in consumption goods production and countries with a larger difference in the intensities experience a larger decline in the relative price of investment and a larger increase in the investment rate. The value of the tradable-intensities in investment goods production determines the transition path for capital in our model. The transition path for TFP is mainly determined by the value of the tradable-intensities in consumption goods production.

We provide a fast computational method for solving multicountry trade models with large state spaces. The state variables in our model include capital stocks as well as net foreign asset positions. Our algorithm iterates on a subset of prices using excess demand equations and delivers the entire transition path for 44 countries in less than two hours on a standard computer (see also Alvarez and Lucas, 2007). Our algorithm uses gradient-free updating rules that are computationally less demanding than the nonlinear solvers used in recent dynamic models of trade (see, e.g., Eaton, Kortum, Neiman, and Romalis, 2016; Kehoe, Ruhl, and Steinberg, 2016).

Our paper is related to three papers on multi-country models with capital accumulation: Alvarez and Lucas (2017), Eaton, Kortum, Neiman, and Romalis (2016), and Anderson, Larch, and Yotov (2015).[1] Alvarez and Lucas (2017) approximate the dynamics in a model with period-by-period balanced trade by linearizing around the counterfactual steady state. Our computational method provides an exact dynamic path. The linear approximation might be inaccurate for computing transitional dynamics in cases of large trade liberalizations. For instance, we find that even with balanced trade the dynamic gain increases exponentially with reductions in trade frictions. Furthermore, in our model, endogenous trade imbalances and capital accumulation imply that the counterfactual steady state cannot be computed without solving for the transitional dynamics; both have to be solved simultaneously.

Eaton, Kortum, Neiman, and Romalis (2016) examine the collapse of trade during the recent global recession. They quantify the roles of different shocks via counterfactuals by

[1]Baldwin (1992) and Alessandria, Choi, and Ruhl (2014) study welfare gains in two-country models with capital accumulation and balanced trade (see also Brooks and Pujolas, 2016).

solving the planner's problem. In their computation, each country's share in world consumption expenditures is the same in the benchmark and in the counterfactual since the Pareto weight for each country is its share. Instead, we solve for the competitive equilibrium and find that each country's share changes in the counterfactual. For example, Bulgaria's share increases by 30 percent across steady states, whereas the U.S.'s share decreases.

Anderson, Larch, and Yotov (2015) compute transitional dynamics in a model where the relative price of investment and the investment rate do not depend on trade frictions. The investment rate can be computed once and for all as a constant pinned down by the structural parameters. The transition path can then be computed as a solution to a sequence of static problems. In our model, the current allocations and prices depend on the entire path of prices and trade frictions. Hence, we have to simultaneously solve a system of second-order, non-linear difference equations.

The investment rate increases in our model in response to the trade liberalization. This is consistent with the evidence in Wacziarg and Welch (2008), who show the increase in investment rate after trade liberalizations for a sample of 118 countries.

Our paper is also related to recent studies that use sufficient statistics in static models. They measure changes in welfare by changes in income which are completely described by changes in the home trade share (e.g., ACR). In our model with endogenous trade imbalances, changes in the home trade share are not sufficient to characterize the changes in welfare since changes in home trade share do not reflect changes income and consumption is not proportional to income in steady state or along the transition.

The rest of the paper proceeds as follows. Section 2 presents the model with trade imbalances. Section 3 describes the calibration and Section 4 reports the results from the counterfactual exercise. Section 5 explores the role of capital accumulation. Section 6 examines the role of trade imbalances and intensities of tradables. Section 7 concludes.

2 Model

There are I countries indexed by $i = 1, \ldots, I$ and time is discrete, running from $t = 1, \ldots, \infty$. There are three sectors: consumption, investment, and intermediates, denoted by c, x, and m, respectively. Neither consumption goods nor investment goods are tradable. There is a continuum of intermediate *varieties* that are tradable. Trade in intermediate varieties is subject to iceberg costs. (In Appendix F, we enrich our model with more sectors and a complete input-output (IO) structure. Every sector's output is used for intermediate goods and final goods production, and the final product is used for consumption and investment.)

Each country has a representative household that owns the country's primary factors of production, capital and labor. Capital and labor are mobile across sectors within a country but are immobile across countries. The household inelastically supplies capital and labor to domestic firms and purchases consumption and investment goods from the domestic firms. Investment augments the stock of capital.

Households can trade one-period bonds. There is no uncertainty and households have perfect foresight.

2.1 Endowments

The representative household in country i is endowed with a labor force of size L_i in each period, an initial stock of capital, K_{i1}, and an initial net foreign asset (NFA) position, $\mathcal{A}_{i1}$.

2.2 Technology

There is a continuum of varieties in the intermediates sector. Each variety is tradable and is indexed by $v \in [0, 1]$.

Composite good All of the intermediate varieties are combined with constant elasticity to construct a composite intermediate good:

$$M_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties. The term $q_{it}(v)$ is the quantity of variety v used by country i to construct the composite good at time t and M_{it} is the quantity of the composite good available in country i as input.

Varieties Each variety is produced using capital, labor, and the composite good. The technologies for producing each variety are given by

$$Y_{mit}(v) = z_{mi}(v) \left(K_{mit}(v)^{\alpha} L_{mit}(v)^{1-\alpha} \right)^{\nu_{mi}} M_{mit}(v)^{1-\nu_{mi}}.$$

The term $M_{mit}(v)$ denotes the quantity of the composite good used by country i as an input to produce $Y_{mit}(v)$ units of variety v, while $K_{mit}(v)$ and $L_{mit}(v)$ denote the quantities of capital and labor used. The parameter $\nu_{mi} \in [0, 1]$ denotes the share of value added in total output in country i and α denotes capital's share in value added. These parameters are constant over time.

The term $z_{mi}(v)$ denotes country i's productivity for producing variety v. Following Eaton and Kortum (2002), the productivity draw comes from independent Fréchet distributions with shape parameter θ and country-specific scale parameter T_{mi}, for $i = 1, 2, \ldots, I$. The c.d.f. for productivity draws in country i is $F_{mi}(z) = \exp(-T_{mi}z^{-\theta})$.

Consumption good Each country produces a final consumption good using capital, labor, and intermediates according to

$$Y_{cit} = A_{ci}\left(K_{cit}^{\alpha}L_{cit}^{1-\alpha}\right)^{\nu_{ci}} M_{cit}^{1-\nu_{ci}}.$$

The terms K_{cit}, L_{cit}, and M_{cit} denote the quantities of capital, labor, and the composite good used by country i to produce Y_{cit} units of consumption at time t. The parameter ν_{ci} is constant over time. The term A_{ci} captures country i's productivity in the consumption goods sector.

Investment good Each country produces an investment good using capital, labor, and intermediates according to

$$Y_{xit} = A_{xi}\left(K_{xit}^{\alpha}L_{xit}^{1-\alpha}\right)^{\nu_{xi}} M_{xit}^{1-\nu_{xi}}.$$

The terms K_{xit}, L_{xit}, and M_{xit} denote the quantities of capital, labor, and the composite good used by country i to produce Y_{xi} units of investment at time t. The parameter ν_{xi} is constant over time. The term A_{xi} captures country i's productivity in the investment goods sector. Note that $1 - \nu_{xi}$ denotes the intensity of tradables in investment goods, so that when $\nu_{xi} < \nu_{ci}$ investment goods production is more tradables-intensive than consumption goods production.

Capital accumulation The representative household enters period t with K_{it} units of capital, which depreciates at the rate δ. Investment, X_{it}, adds to the stock of capital subject to an adjustment cost.

$$K_{it+1} = (1-\delta)K_{it} + \chi X_{it}^{\lambda}K_{it}^{1-\lambda},$$

where χ reflects the marginal efficiency of investment, and λ is the elasticity of capital accumulation with respect to investment.[2] For convenience, we work with investment:

$$X_{it} = \Phi(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\lambda}} (K_{it+1} - (1-\delta)K_{it})^{\frac{1}{\lambda}} K_{it}^{\frac{\lambda-1}{\lambda}}.$$

Net foreign asset accumulation The household can borrow or lend to the rest of the world by trading one-period bonds; let B_{it} denote the net purchases of bonds by country i and q_t denote the world interest rate on bonds at time t. The representative household enters period t with a net foreign asset (NFA) position $\mathcal{A}_{it}$. If $\mathcal{A}_{it} < 0$ then country i is indebted at time t. The NFA position evolves according to

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + B_{it}.$$

We assume that all debts are eventually paid off. Countries that borrow in the short run to finance trade deficits will have to pay off the debts in the long run via perpetual trade surpluses. Each country's current account balance, B_{it}, equals net exports plus net foreign income on assets:

$$B_{it} = P_{mit} \left(Y_{mit} - M_{it}\right) + q_t \mathcal{A}_{it},$$

where $P_{mit} M_{it}$ is the total expenditure on intermediates including imported intermediates and $P_{mit} Y_{mit}$ is total sales including exports.

Budget constraint The representative household earns a rental rate r_{it} on capital and a wage rate w_{it} on labor. If the household has a positive NFA position at time t, then net-foreign income, $q_t \mathcal{A}_{it}$, is positive. Otherwise resources are used to pay off existing liabilities. The household purchases consumption at the price P_{cit} and purchases investment at the price P_{xit}. The budget constraint is given by

$$P_{cit} C_{it} + P_{xit} X_{it} + B_{it} = r_{it} K_{it} + w_{it} L_i + q_t \mathcal{A}_{it}.$$

[2]Without adjustment costs, the household's choice of bonds and capital in our model is indeterminate. The adjustment cost specification implies that, from each household's perspective, the rate of return on investment depends on the quantity of investment and the household chooses a unique portfolio.

2.3 Trade

International trade is subject to frictions that take the iceberg form. Country i must purchase $d_{ij} \geq 1$ units of an intermediate variety from country j in order for one unit to arrive; $d_{ij} - 1$ units melt away in transit. As a normalization, we assume that $d_{ii} = 1$ for all i.

2.4 Preferences

The representative household's lifetime utility is given by

$$\sum_{t=1}^{\infty} \beta^{t-1} \frac{(C_{it}/L_i)^{1-1/\sigma}}{1 - 1/\sigma},$$

where C_{it}/L_i is consumption per worker in country i at time t, $\beta \in (0, 1)$ denotes the period discount factor, and σ denotes the intertemporal elasticity of substitution. Both parameters are constant across countries and over time.

2.5 Equilibrium

At each point in time, we take world GDP as the numéraire: $\sum_i r_{it} K_{it} + w_{it} L_i = 1$ for all t. That is, all prices are expressed in units of current world GDP.

A competitive equilibrium satisfies the following conditions: (i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for capital accumulation; (ii) taking prices as given, firms maximize profits subject to the available technologies; (iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade frictions; and (iv) all markets clear. We describe each equilibrium condition in more detail in Appendix A.

In addition to the above equilibrium conditions, a steady state is characterized by a balanced current account and time-invariant consumption, output, capital stock, and net foreign asset position. In the steady state, net foreign income offsets the trade imbalance.

2.6 Welfare Gains

We compute transition paths for several counterfactuals starting from an initial steady state to a final steady state. We measure the resulting changes in welfare using consumption equivalent units as in Lucas (1987). Let $c_i \equiv C_i/L_i$ denote consumption per worker in country i. The *dynamic gain* in country i is measured by λ_i^{dyn} that solves:

$$\sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(\left(1+\frac{\lambda_i^{dyn}}{100}\right) c_i^{\star}\right)^{1-1/\sigma}}{1-1/\sigma} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(\tilde{c}_{it}\right)^{1-1/\sigma}}{1-1/\sigma}, \tag{1}$$

where $c_i^{\star}$ is the initial steady state and $\tilde{c}_{it}$ is consumption at time t in the counterfactual.

The transition path for consumption depends on the path for income. We denote real income per worker as $y_{it} \equiv \frac{r_{it}K_{it}+w_{it}L_{it}}{P_{cit}L_{it}}$ and capital-labor ratio as $k_{it} \equiv \frac{K_{it}}{L_i}$. In Appendix B we show that

$$y_{it} \propto \underbrace{\left(\frac{A_{ci}}{B_{ci}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{iit}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{1-\nu_{ci}}{\nu_{mi}}}}_{\text{Measured TFP}} (k_{it})^{\alpha}, \tag{2}$$

where $B_{ci} = (\alpha\nu_{ci})^{-\alpha\nu_{ci}} ((1-\alpha)\nu_{ci})^{-(1-\alpha)\nu_{ci}} (1-\nu_{ci})^{-(1-\nu_{ci})}$ and B_{mi} is defined analogously by replacing ν_{ci} with ν_{mi}. In equation (2), the capital-labor ratio is endogenous and is also a function of the home trade share.

Channels for the gains from trade Trade liberalization affects the dynamic gain in our model through two channels.

1. Trade liberalization results in an immediate and permanent drop in the home trade shares and, hence, higher measured TFP on impact. The higher measured TFP increases GDP and affects the consumption path.

2. Trade liberalization also increases the rate of capital accumulation due to the increase in TFP and decrease in the relative price of investment.

    - The increase in TFP yields a higher marginal product of capital (MPK), which affects capital accumulation and, hence, income and consumption.
    - Trade liberalization also reduces the prices of intermediate varieties. If investment goods production is more intensive in such tradable inputs ($\nu_{xi} < \nu_{ci}$), the relative price of investment goods declines. A lower relative price of investment makes it feasible to allocate a larger share of income to investment without sacrificing consumption. Hence, the investment rate and capital-labor ratio increase, affecting income and consumption along the transition path.

The rate of accumulation of capital depends on the relative price of investment:

$$\frac{P_{xit}}{P_{cit}} \propto \left(\frac{B_{xi}}{B_{ci}}\right)\left(\frac{A_{ci}}{A_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{\nu_{xi}-\nu_{ci}}{\nu_{mi}}}, \tag{3}$$

where $B_{xi} = (\alpha\nu_{xi})^{-\alpha\nu_{xi}}\left((1-\alpha)\nu_{xi}\right)^{-(1-\alpha)\nu_{xi}}(1-\nu_{xi})^{-(1-\nu_{xi})}$.

Dynamics The dynamics are governed by two intertemporal Euler equations associated with the one-period bond and capital:

$$\frac{c_{it+1}}{c_{it}} = \beta^{\sigma}\left(\frac{1+q_{t+1}}{P_{cit+1}/P_{cit}}\right)^{\sigma} \tag{4}$$

and

$$\frac{c_{it+1}}{c_{it}} = \beta^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{ixt+1}} - \Phi_2(k_{it+2}, k_{it+1})}{\Phi_1(k_{it+1}, k_{it})}\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}, \tag{5}$$

where $\Phi_1(\cdot,\cdot)$ and $\Phi_2(\cdot,\cdot)$ denote the first derivatives of the adjustment-cost function with respect to the first and second arguments, respectively:

$$\Phi_1(k', k) = \left(\frac{1}{\chi}\right)^{\frac{1}{\lambda}}\left(\frac{1}{\lambda}\right)\left(\frac{k'}{k} - (1-\delta)\right)^{\frac{1-\lambda}{\lambda}}$$
$$\Phi_2(k', k) = \left(\frac{1}{\chi}\right)^{\frac{1}{\lambda}}\left(\frac{1}{\lambda}\right)\left(\frac{k'}{k} - (1-\delta)\right)^{\frac{1-\lambda}{\lambda}}\left((\lambda-1)\frac{k'}{k} - \lambda(1-\delta)\right),$$

where the prime notation denotes the next period's value.

The dynamics are pinned down by the solution to a system of I simultaneous, second-order, nonlinear difference equations. Note that the dynamics of capital in country i depend on the capital stocks in all other countries due to trade. The Euler equations also reveal that a change in trade friction for any country at any point in time affects the dynamic path of all countries.

3 Calibration

We calibrate the parameters of our model to match several observations in 2014. We assume that the world is in steady state in 2014. Our data cover 44 countries (more precisely, 43 countries plus a rest-of-the-world aggregate). Table C.1 in Appendix C provides a list of the countries. The primary data sources include version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)) and the World Input-Output Database (Timmer, Dietzenbacher, Los, and de Vries, 2015; Timmer, Los, Stehrer, and de Vries, 2016, (WIOD)). More details about the data are provided in Appendix C.

Initial steady state With endogenous trade imbalances, the transition path and the steady state are determined jointly. To compute the initial steady state, we use two properties of steady state to specify the steady-state values for the NFA positions, $\mathcal{A}_{i1}$, in every country: (i) the world interest rate is $q = 1/\beta - 1$ and (ii) the current account is balanced. These two properties imply that $\mathcal{A}_{i1}$ satisfy $NX_i = -q\mathcal{A}_i$, i.e., the net exports in steady state, NX_i, is offset by net foreign income. We choose net foreign income so that the trade deficits in steady state are those observed in 2014. The initial steady state is then characterized by a set of nonlinear equations; see Table A.2 in Appendix A.

3.1 Common parameters

The values for the common parameters are reported in Table 1. We use recent estimates of the trade elasticity by Simonovska and Waugh (2014) and set $\theta = 4$. We set $\eta = 2$, which satisfies the condition: $1 + \frac{1}{\theta}(1 - \eta) > 0$. This value plays no quantitative role in our results.

In line with the literature, we set the share of capital in value added to $\alpha = 0.33$ (Gollin, 2002), the discount factor to $\beta = 0.96$, so that the steady-state real interest rate is about 4 percent, and the intertemporal elasticity of substitution to $\sigma = 0.5$.

The rate of depreciation for capital is set to $\delta = 0.06$. The elasticity of capital accumulation with respect to investment, λ, is set to 0.76.[3] The marginal efficiency of investment is set to $\chi = \delta^{1-\lambda}$ so that there are no adjustment costs in the steady state (i.e., $X_i = \delta K_i$).

3.2 Country-specific parameters

As noted earlier, with $q = 1/\beta - 1$, we choose $\mathcal{A}_{i1}$ to be consistent with the observed trade imbalances in each country in 2014; the current account balance is zero.

[3]Eaton, Kortum, Neiman, and Romalis (2016) calibrate this value to be 0.5 for investment in structures and 0.55 for investment in equipment in a model that uses quarterly data. First, we compute the average between the two, as we have only one investment good. Second, since we use annual data and their quarterly values likely overestimate the annual adjustment cost, we take the midpoint between the average of their estimates and 1, where $\lambda = 1$ corresponds to no adjustment costs.

Table 1: Common parameters

Trade elasticity	θ	4
Elasticity of substitution between intermediate varieties	η	2
Capital's share in value added	α	0.33
Discount factor	β	0.96
Intertemporal elasticity of substitution	σ	0.5
Depreciation rate for capital	δ	0.06
Marginal efficiency of investment	χ	0.28
Adjustment cost elasticity	λ	0.76

We calibrate intermediate-input intensities ν_{mi}, ν_{xi}, and ν_{ci} using data from WIOD. For ν_{mi} we compute the ratio of value added to gross output for non-durable goods production in each country, which covers two-digit categories 01-28 in revision 3 of the International Standard Industrial Classification of All Economic Activities (ISIC). To compute ν_{xi} we compute the ratio of value added to gross output for durable goods (ISIC categories 29-35) and construction (ISIC category 45). Finally, we compute the remainder of value added and gross output in each country for those sectors that are not accounted for by sectors m and x to obtain values for ν_{ci} in each country. The cross-country heterogeneity in the intensities are illustrated in Figure 1. The cross-country averages for ν_{mi}, ν_{xi}, and ν_{ci} are 0.33, 0.33, and 0.56, respectively.

We set the workforce, L_i, equal to the employment in country i in 2014, documented in PWT. The remaining parameters A_{ci}, T_{mi}, A_{xi}, and d_{ij}, for $(i, j) = 1, \ldots, I$, are not directly observable. We infer these by linking steady-state relationships of the model to observables.

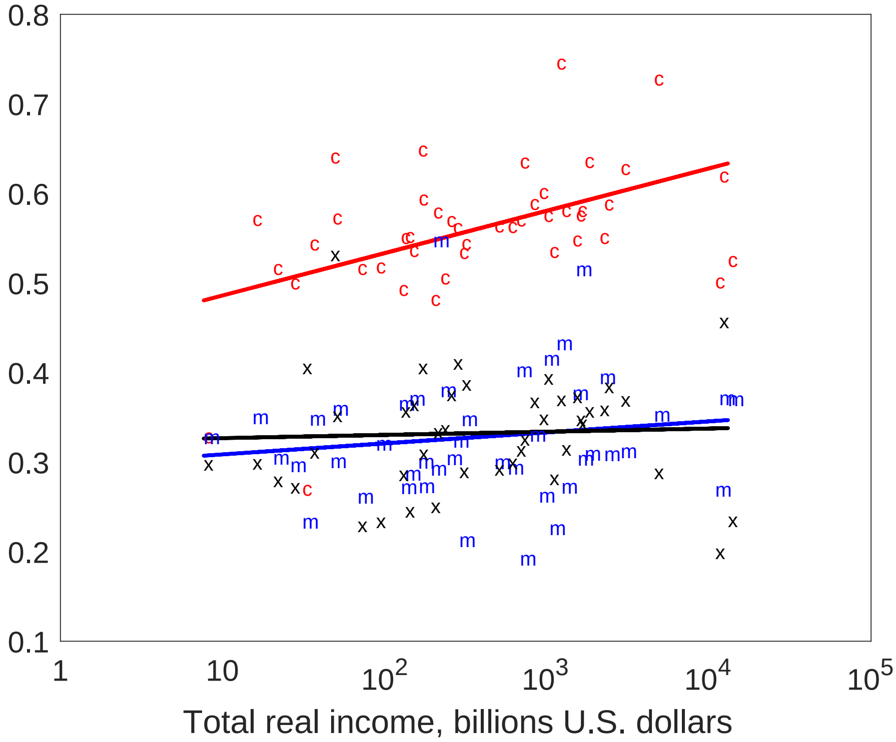
The equilibrium structure relates the unobserved trade frictions between any two countries to the ratio of intermediate goods prices in the two countries and the trade shares:

$$\frac{\pi_{ij}}{\pi_{jj}} = \left(\frac{P_{mj}}{P_{mi}}\right)^{-\theta} d_{ij}^{-\theta}. \tag{6}$$

Appendix C describes how we construct the empirical counterparts to prices and trade shares. For observations in which $\pi_{ij} = 0$, we set $d_{ij} = 10^8$. We also set $d_{ij} = 1$ if the inferred value of trade cost is less than 1.

Lastly, we use three structural relationships to pin down the productivity parameters

Figure 1: Ratio of value added to gross output in each sector



Notes: The letters c, x, and m in each scatter plot denote the consumption, investment, and intermediate sectors, respectively. Horizontal axis–Total real GDP data for 2014.

A_{ci}, T_{mi}, and A_{xi}:

$$\frac{P_{ci}}{P_{mi}} \propto \left(\frac{B_{ci}}{A_{ci}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{\nu_{ci}}{\nu_{mi}}} \tag{7}$$

$$\frac{P_{xi}}{P_{mi}} \propto \left(\frac{B_{xi}}{A_{xi}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{\nu_{xi}}{\nu_{mi}}} \tag{8}$$

$$y_i \propto \left(\frac{A_{ci}}{B_{ci}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{1-\nu_{ci}}{\nu_{mi}}} (k_i)^{\alpha}. \tag{9}$$

As noted earlier, the terms B_{ci}, B_{mi}, and B_{xi} are country-specific constants that depend on $\alpha, \nu_{ci}, \nu_{mi}$, and ν_{xi}. Equations (7)–(9) are derived in Appendix B. The three equations relate observables—the price of consumption relative to intermediates, the price of investment relative to intermediates, income per worker, capital stocks, and home trade shares—to the unknown productivity parameters. We normalize $A_{ci} = T_{mi} = A_{xi} = 1$ for the United States. For each country i, system (7)–(9) yields three nonlinear equations with three unknowns: A_{ci}, T_{mi}, and A_{xi}. Information about constructing the empirical counterparts to $P_{ci}, P_{mi}, P_{xi}, y_i, K_i$, and π_{ii} is in Appendix C.

These equations are quite intuitive. The expression for income per worker provides a measure of aggregate productivity across all sectors: Higher income per worker is associated with higher productivity levels, on average. The expressions for relative prices boil down to two components. The first term reflects something akin to the Balassa-Samuelson effect: All else equal, a higher price of capital relative to intermediates suggests a low productivity in capital goods sector relative to intermediate goods sector. In our setup, the measured productivity for intermediates is endogenous, reflecting the degree of specialization as captured by the home trade share. The second term reflects the relative intensity of intermediate inputs. If measured productivity is high in intermediates, then the price of intermediates is relatively low and the sector that uses intermediates more intensively will have a lower relative price. In our calibration, as Figure 1 illustrates, the intermediates are more intensively used in the capital goods sector, that is, $\nu_{xi} < \nu_{ci}$.

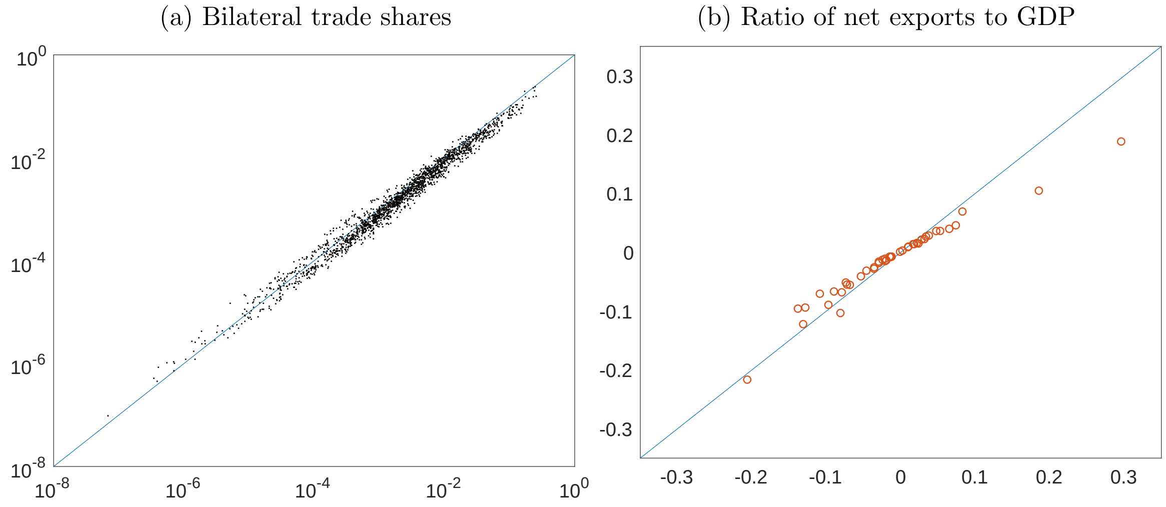
3.3 Model fit

Our model consists of 2021 unobservable country-specific parameters: $I(I-1) = 1892$ bilateral trade frictions, $(I-1) = 43$ consumption-good productivity terms, $(I-1) = 43$ investment-good productivity terms, and $(I-1) = 43$ intermediate-goods productivity terms.

Calibration of the country-specific parameters uses a total of 2107 data points. The trade frictions use up $I(I-1) = 1892$ data points for bilateral trade shares and $(I-1) = 43$ for the ratio of absolute prices of intermediates. The productivity parameters use up $(I-1) = 43$ data points for the price of consumption relative to intermediates, $(I-1) = 43$ data points for the price of investment relative to intermediates, $(I-1) = 43$ data points for income per worker, and $(I-1) = 43$ data points for capital stocks.

The model matches the targeted data well. The correlation between model and data is 0.98 for bilateral trade shares (see Figure 2a). The correlation is 0.99 for the absolute price of intermediates, 0.95 for income per worker, 0.98 for the price of consumption relative to intermediates, and 0.98 for the price of investment relative to intermediates. Our model also matches the targeted ratio of net exports to GDP; the correlation is 0.94 (see Figure 2b).

Figure 2: Model fit: Bilateral trade shares and net exports to GDP



Notes: Horizontal axis–Data; Vertical axis–model.

We use prices of consumption and investment, relative to intermediates, in our calibration. The correlation between the model and the data is 0.98 for the absolute price of consumption and 0.98 for the absolute price of investment. The correlation for the price of investment relative to consumption is 1.00.

Untargeted moments The correlation between the model and the data on capital-labor ratios is 0.71. In both the model and the data, the nominal investment rate is uncorrelated with the level of income per worker. The cross-country average nominal investment rate, $\frac{P_x X}{wL+rK}$, is 17.4 percent in the model and is 23.3 percent in the data.

4 Counterfactuals

In this section, we implement a counterfactual trade liberalization via an unanticipated, uniform, and permanent reduction in trade frictions. The world begins in the calibrated steady state. At the beginning of period $t = 1$, trade frictions fall uniformly by 20 percent in all countries. This amounts to reducing $d_{ij} - 1$ by 20 percent for each country pair i, j. All other parameters are fixed at their calibrated values. (In Appendix E, we consider a non-uniform trade liberalization by decomposing trade frictions into a gravity component, i.e., driven by geography, and a policy component that is heterogeneous across countries, and then removing all asymmetries in trade frictions by reducing the policy component in each country to the same value.)

4.1 Computing the counterfactual transition path and steady state

The main challenge in solving dynamic multicountry trade models is the curse of dimensionality. Computing the dynamic paths requires solving intertemporal Euler equations and each one of our Euler equations is a second order, nonlinear difference equation. In closed economies or two-country models, recursive methods such as value function iteration or policy function iteration can be employed efficiently by discretizing the state space for capital stocks in each country. However, in our world with 44 countries, there are two state variables and n discrete values for each would imply $n^{44} \times n^{44}$ grid points in the state space. An alternative is to use recent advances in shooting algorithms that involve iterating on guesses for the entire path of state variables in every country. Each iteration, however, involves computing gradients to update the entire path. With T periods and 44 countries, the updates require $44 \times T$ gradients for each variable, and each gradient requires solving the entire model.

Our method iterates on prices and investment rates. We use excess demands to determine the size and direction of the change in prices and investment rates in each iteration. We bypass the costly computation of gradients and can compute the entire transition path in less than two hours on a standard computer.

To compute the counterfactual transition path and the counterfactual steady state, we first reduce the infinite horizon problem to a finite horizon model with $t = 1, \ldots, T$ periods. We make T sufficiently large to ensure convergence to a new steady state; $T = 150$ proved sufficient in our computations.

We start with a guess: The terminal NFA position $\mathcal{A}_{iT+1} = 0$, for all i. We then guess the entire sequences of nominal investment rates, $\rho_{it} = \frac{P_{xit}X_{it}}{w_{it}L_{it}+r_{it}K_{it}}$, and wages for every country, and one sequence of world interest rates. Taking the nominal investment rate as given we iterate over wages and the world interest rate using excess demand equations. The wages and the world interest rate help us recover all other prices and trade shares from first-order conditions and a subset of market-clearing conditions. We use deviations from the balance of payments identity—net purchases of bonds equals net exports plus net foreign income—and trade balance at the world level to update the sequences of wages in every country and the world interest rate simultaneously. Once we find sequences that satisfy balance of payments, we check whether the Euler equation for investment in capital is satisfied. We use deviations from the Euler equation to update the nominal investment rate in every country at every point in time simultaneously. Using the transition path of the NFA position, we update the terminal $\mathcal{A}_{iT+1}$ by setting it to $\mathcal{A}_{it}$ where t is some period close to but less than T. We continue this procedure until we reach a fixed point in the sequence of nominal investment rates and the steady-state NFA position. Appendix D describes our solution method in more detail. Our method is also valid for the environment with the complete IO structure (Appendix F) and for non-uniform trade liberalizations (Appendix E).

The presence of both capital and bonds introduces a unique challenge in computing transitional dynamics. To see why, consider a model with one-period bonds but no capital accumulation, as in Reyes-Heroles (2016). In such an environment, a one-time change to trade frictions yields a one-time change to the counterfactual steady state, since all factors immediately adjust on impact. In a model with capital accumulation, capital adjusts to its new steady state gradually, because of both diminishing returns to investment and adjustment costs. As each country's capital stock adjusts, current accounts respond in order to equalize MPKs across countries. The steady-state NFA position depends on the current account dynamics since debts are perpetually served in the new steady state and the number of periods it takes for the economy to reach its new steady state is endogenous. Thus, the terminal counterfactual steady-state cannot be determined independently from the initial condition and the transition. Put differently, in models without capital one can choose an arbitrary period when the economy reaches the steady state, but we cannot. Hence, half-life for capital accumulation is endogenous in our model.[4]

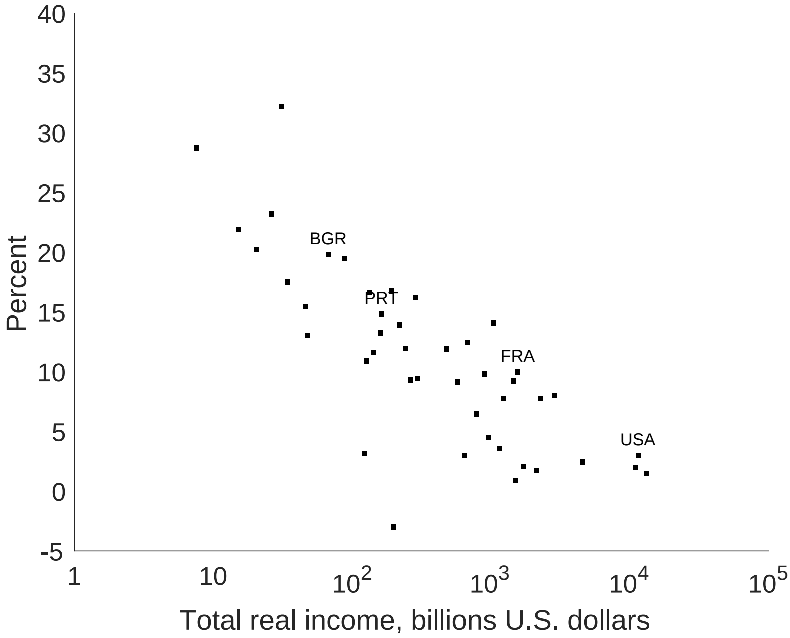
4.2 Dynamic gains from trade

As noted earlier, the dynamic gain for country i, λ_i^{dyn}, is given by equation (1). Figure 3 illustrates the dynamic gains from a 20 percent reduction in trade costs for the 44 countries in our sample. Throughout the remainder of the paper, we not only use scatter plots as in

[4]Eaton, Kortum, Neiman, and Romalis (2016) use the "hat algebra" approach to solve for *changes* in endogenous variables; Zylkin (2016) uses a similar approach to study the dynamic effects of China's integration into the world economy; Caliendo, Dvorkin, and Parro (2015) also use hat algebra to study the dynamic effects of increased competition from China on U.S. labor markets in a trade model without capital. In this method, the computation of the counterfactual can proceed without knowing several structural parameters and initial levels of endogenous variables. Our approach is different: (i) we solve for the counterfactual transition path in *levels*, (ii) our computation requires knowledge of all structural parameters, and (iii) we ensure that the baseline predictions for endogenous variables in our model are consistent with the data before computing the counterfactual.

Figure 3, but also use four countries to highlight our results: Bulgaria, Portugal, France, and the United States. These four countries provide a representative sample of gains and of size, measured by total real GDP.

Figure 3: Distribution of gains from trade

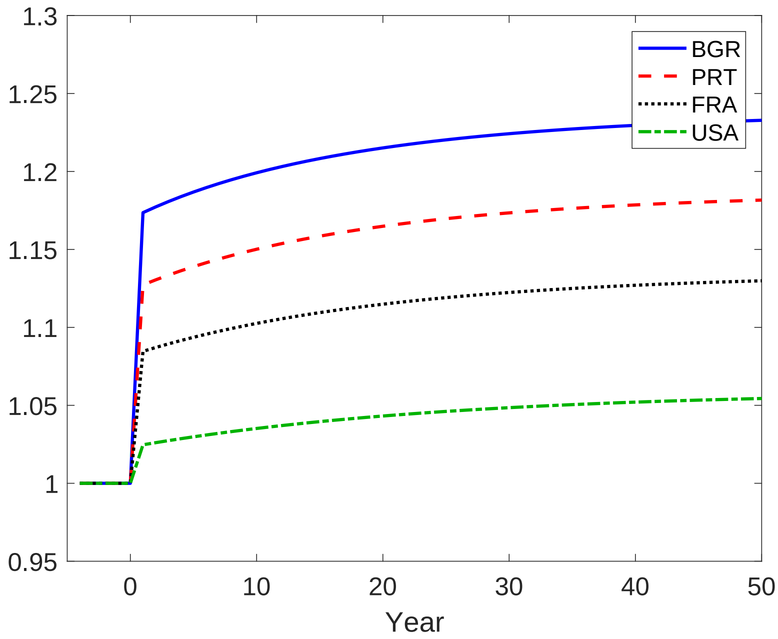


Notes: Horizontal axis–Total real GDP data for 2014. Vertical axis–Dynamic gains (percent) following a uniform, unanticipated, permanent, 20 percent trade liberalization. The gain for Norway is negative. This is due to its large negative net foreign asset position in the initial steady state. See details in Section 6.1.

The gains from trade vary substantially across countries: The gain for the United States is 4.4 percent while the gain for Bulgaria is 21 percent. The gains are smaller for large countries, similar to the findings in Waugh and Ravikumar (2016) and Waugh (2010). Since the size of liberalization is the same for all countries, the implied elasticities—the percent increase in welfare due to the percent decrease in trade cost—are also different across countries. For this counterfactual experiment, the elasticity is roughly 0.22 for the United States and 1.06 for Bulgaria. (In Appendix F, we compare these welfare gains to those in a model with more sectors and a complete IO structure. We find that the two welfare gains are highly correlated, but the gain in the IO model tends to be lower.)

The consumption paths that generate the gains are illustrated in Figure 4 for the four countries. Bulgaria, for instance, not only experiences a larger increase in consumption immediately after the trade liberalization but also ends up with a larger increase in consumption across steady states, relative to the United States.

Figure 4: Transition path for consumption



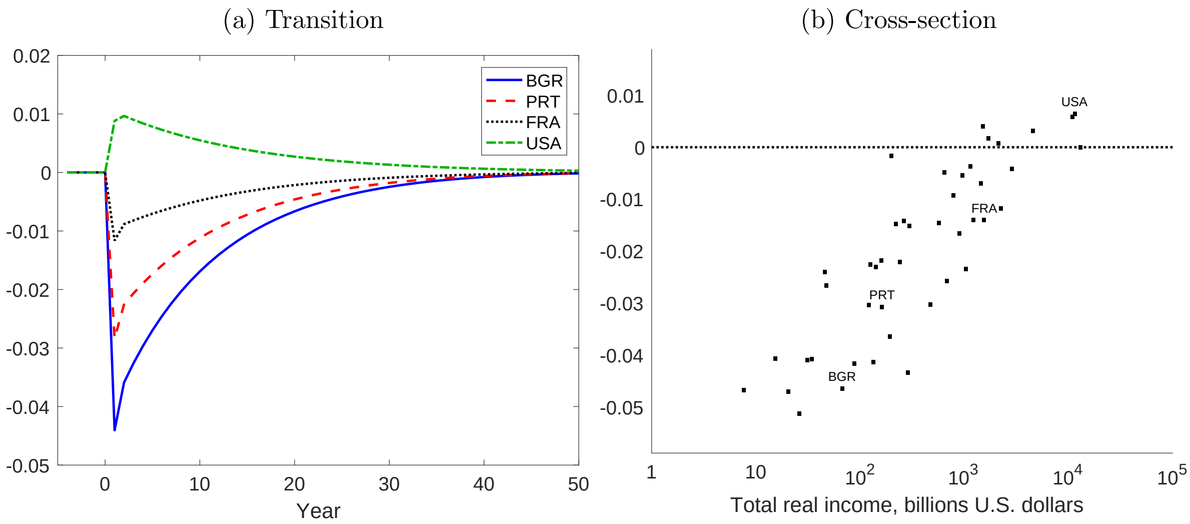
Notes: Transitions following a uniform, unanticipated, permanent, 20 percent trade liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1.

The manner in which the consumption path is financed differs across countries. Figure 5 illustrates the current accounts. Recall that all countries start from an initial steady state of zero current account balance. United States accumulates a current account surplus immediately after the liberalization, whereas Bulgaria has a current account deficit. The current account balance is positively correlated with country size. Financial resources flowing from large countries to small countries help boost the consumption in small countries.

As noted in Section 2.6, trade liberalization reduces each country's home trade share immediately, increasing each country's TFP (see equation 2) and reducing the relative price of investment. See Figure 6.

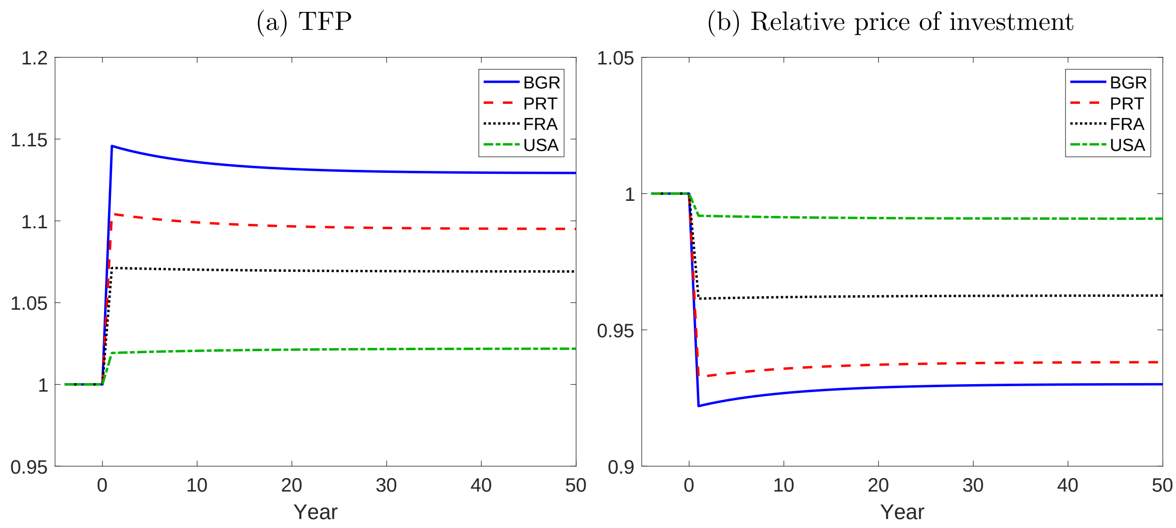
The immediate increase in TFP increases each country's output; capital does not change on impact. See Figure 7. Higher output makes more consumption and investment feasible. Optimal allocation of the higher output to consumption and investment determines the dynamics and is governed by the relative price of investment and the return to capital, as revealed by the Euler equation (5). Investment increases by more than consumption because (i) the relative price of investment decreases and (ii) higher TFP causes MPK to increase.

Figure 5: Ratio of current account to GDP



Notes: Results following a uniform, unanticipated, permanent, 20 percent trade liberalization. The current account balance is zero in the initial steady state. Panel (a): The liberalization occurs in period 1. Panel (b): Ratio of current account to GDP, computed in period 1. Horizontal axis–Total real GDP data for 2014.

Figure 6: Transition path for TFP and Relative price of investment

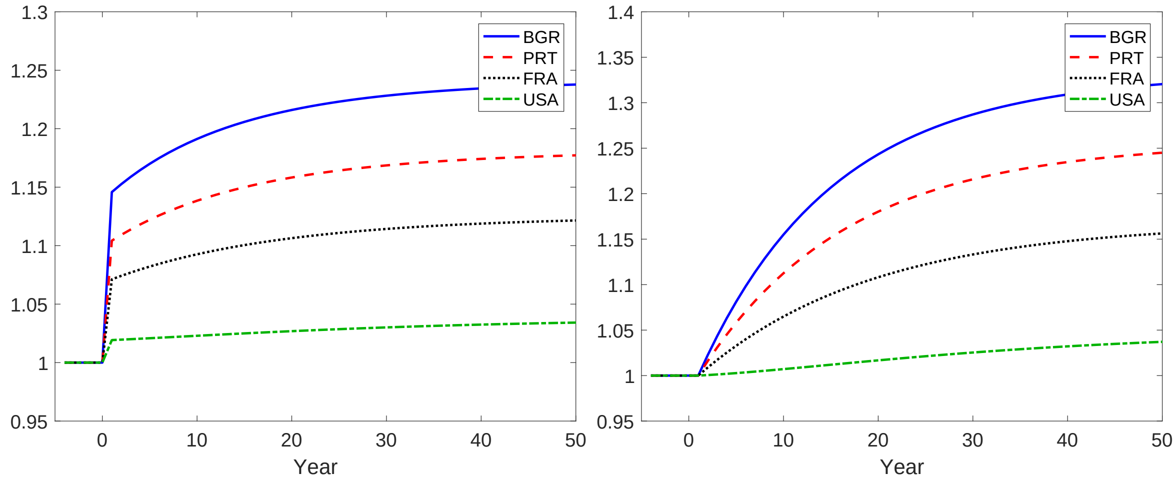


Notes: Transitions following a uniform, unanticipated, permanent, 20 percent trade liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1.

As capital accumulates, output continues to increase. Recall that the increase in output on impact is entirely due to TFP, whereas the increase in output after the initial period is driven entirely by capital accumulation.

Figure 7: Transition path for income per worker and capital

(a) Income per worker (b) Capital-labor ratio



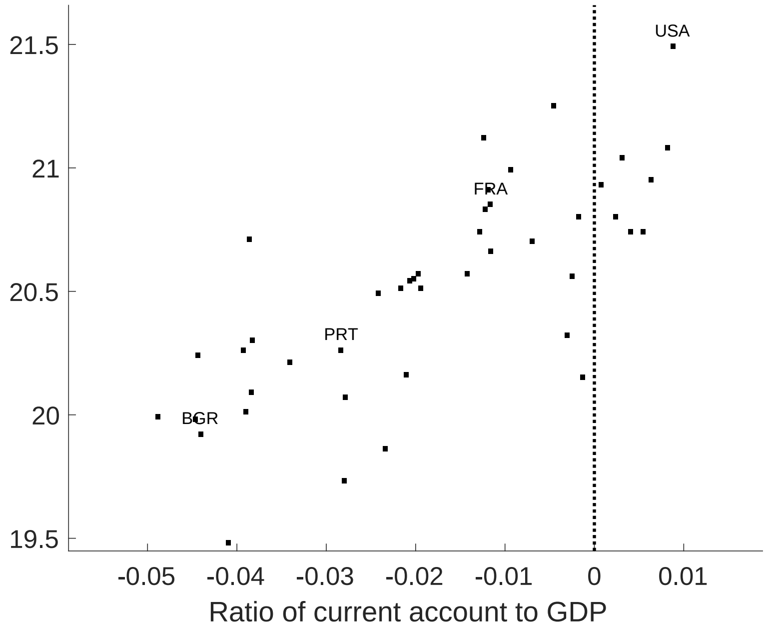
Notes: Transitions following a uniform, unanticipated, permanent, 20 percent trade liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1.

With a frictionless bond market, MPKs are equalized across countries and resources flow to countries that experience a larger increase in TFP. These countries run a current account deficit in the short run and use it to finance increases in consumption and investment that exceed increases in output (e.g., Bulgaria, Portugal, and France). In the new steady state the current account is balanced, but countries that accumulate debt along the transition have to run trade surpluses to service the debt. In general, small countries run current account deficits and large countries run current account surpluses in the short run.

Half life The behavior of trade imbalances also reveals a pattern in the rates of capital accumulation. Figure 8 illustrates that the half-life for capital accumulation—the number of periods it takes for the capital stock to reach the midpoint between the initial and counterfactual steady-state values—varies with trade deficits.

Countries with larger trade deficits in the short run have lower half lives, i.e., they accumulate capital faster. Bulgaria closes 50 percent of the gap between its two steady-state values of capital in roughly 20 years, whereas it takes 22 years for the U.S.

Figure 8: Half-life for capital



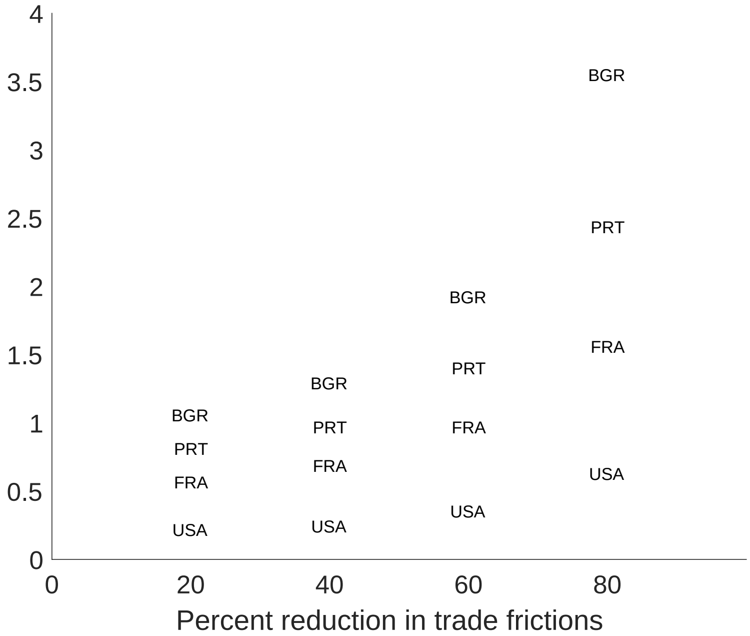
Notes: Half-life for a uniform, unanticipated, permanent, 20 percent trade liberalization. The liberalization occurs in period 1. Horizontal axis–Ratio of current account to GDP, computed in period 1. Vertical axis–Half-life for capital, computed as the number of periods it takes for the capital stock to reach the midpoint between the initial and counterfactual steady-state values.

Nonlinear gains Welfare gains from trade are nonlinear with the size of the trade liberalization. To illustrate these non-linearities, we examine the elasticity of gains, computed as the absolute value of the percent change in welfare divided by the percent change in trade-weighted barriers. The trade-weighted barriers are computed as

$$\bar{d}_i = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{I} TRD_{ji} d_{ji}}{\sum_{\substack{j=1 \\ j \neq i}}^{I} TRD_{ji}} \tag{10}$$

Figure 9 shows the elasticity of gains w.r.t. the reduction in trade costs for Bulgaria, Portugal, France, and the United States, for 20, 40, 60, and 80 percent trade cost reductions. The gains increase exponentially with the size of the liberalization, and the increase is larger for small countries. The elasticity for Bulgaria ranges from 1.06 for a 20 percent trade liberalization to 3.56 for an 80 percent liberalization. The corresponding range for the U.S. is 0.22 to 0.63. (In Appendix E, we show that even for the case of non-uniform trade liberalization the gains are lower for larger countries.)

Figure 9: Elasticity of dynamic gains



Notes: The elasticity is computed as the absolute value of percent change in welfare divided by percent change in trade friction.

5 Role of capital accumulation

In this section, we examine the role of capital accumulation in delivering the gains from trade. To illustrate the role, we compute gains holding capital fixed and compare them to the gains that include capital accumulation. We do this in three ways: (i) We use the counterfactual income path from Figure 7a and construct a gain based on the immediate change in income per worker, holding capital fixed, and compare the gain to the dynamic gain in Section 4.2, (ii) we restrict consumption smoothing over time by constructing a variant of our baseline dynamic model and compare the gains holding capital fixed to the steady-state gains in the restricted model, and (iii) we construct a static model, calibrate it, and compute the static gains. In (i) and (ii), the immediate gains are computed using the transition path of a dynamic model, whereas in (iii) the static gains are computed from a stand-alone static model with its own parameters.

5.1 Immediate gains in the baseline model

In the first approach, we exploit the fact that after an unanticipated trade liberalization capital does not change on impact and the changes in TFP are immediate in our baseline dynamic model (see Figure 6). Thus, the change in income on impact captures the immediate, or "static", gain. Our immediate gain calculation is in the same spirit as the static gain computation in the literature (e.g., ACR) since the gain is entirely due to changes in TFP resulting from changes in home trade share. (Because households can save, change in consumption differ from change in income at every point in time.)

Using the counterfactual income path in our dynamic model (Figure 7a), we compute the immediate gain as:

$$1 + \frac{\lambda_i^{immediate}}{100} = \frac{y_{i1}}{y_i^\star}, \tag{11}$$

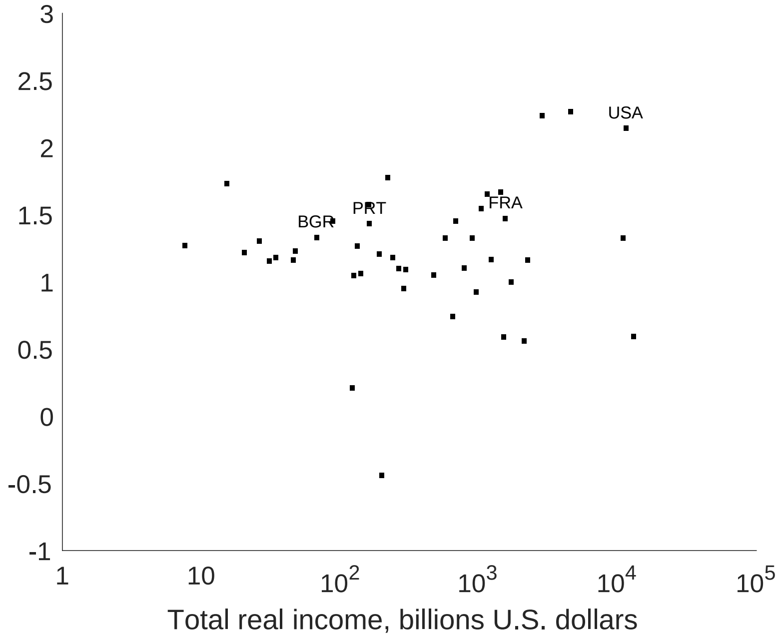
where y_{i1} is the income per worker in country i in period 1 in Figure 7a and $y_i^\star$ is the income per worker in the initial steady state in country i. Note that, conditional on the income path, the immediate gain does not depend on the preference parameters.

The dynamic gains are the same as in our counterfactual in Section 4.2. Figure 10 illustrates the ratio of dynamic gain to immediate gain for each country. On average, the dynamic gain is 35 percent more than the immediate gain. Since capital does not change immediately after liberalization, the additional 35 percent in the dynamic gain is due to capital accumulation and other asset trades over time. The ratio in Figure 10 ranges from -0.32 to almost 2.39. The negative ratio is for Norway whose dynamic gain is negative, as noted earlier in Figure 3, due to its initial large negative foreign asset position.

Several caveats are in order regarding the comparison of dynamic and immediate gains. (i) The dynamic gain is computed using the consumption path while the immediate gain is computed using the income path. (ii) Consumption smoothing in our baseline model is achieved via not only capital accumulation but also asset trades. (iii) Capital accumulation requires not only foregone consumption but also adjustment costs, whereas immediate gains do not include either cost. (iv) Some of the dynamic gains (or losses) are due to the initial NFA position. (v) The immediate gain does not depend on preference parameters, whereas the dynamic gain calculation takes into account the fact that along the transition path the change in consumption from one period to the next is not necessarily equal to the change in utility.

In the next subsection, we construct a variant of our baseline model in Section 2 by imposing restrictions on cross-country trade that help us address the caveats above and

Figure 10: Ratio of dynamic to immediate gains in the baseline model



Notes: Welfare gains are computed following a uniform, unanticipated, permanent, 20 percent trade liberalization. Horizontal axis–Total real GDP data for 2014. Vertical axis–Ratio of dynamic gains to immediate gains using the counterfactual income path in the baseline dynamic model.

assess the quantitative role of capital accumulation.

5.2 Immediate gains in a restricted dynamic model

We study a variant of our baseline model: We impose balanced trade in the initial steady state and in each period in the counterfactual, calibrate the variant, and compute the counterfactual transition path. We will refer to this variant as the "restricted dynamic model."

Balanced trade in the initial steady state eliminates the dependence of dynamic gains on initial trade surpluses or deficits for some countries in Figure 10. Balanced trade in every period allows for capital in each country to be accumulated over time, but prevents consumption smoothing via asset trades across countries helping us isolate the role of capital accumulation. In the restricted dynamic model, income is proportional to consumption *in steady state*. Using the welfare gain in equation (1), it is easy to see that the steady-state gains can be measured using changes in income across steady states:

$$1 + \frac{\lambda_i^{ss}}{100} = \frac{y_i^{\star\star}}{y_i^{\star}}, \tag{12}$$

where λ_i^{ss} is the steady-state gain and $y_i^{\star\star}$ is the income per worker in the counterfactual steady state in country i. Thus, both the immediate gain in (11) and the steady-state gain in (12) use only income and do not depend on the preference parameters and, hence, comparable. We can also compare the steady-state cost of autarky to the immediate cost of autarky using income. (In our baseline model with trade imbalances, we cannot measure "steady-state" gain or cost via change in consumption or via change in income.) Finally, focusing on steady states implies the stock of capital does not change over time, so there is no cost incurred on adjusting capital.

The only difference in the initial steady state between the restricted dynamic model and our baseline model in Section 2 is that each country's trade balance and NFA position are both zero in the restricted dynamic model. The country-specific parameters inferred from the structural relationships (6)–(9) do not depend on the country's trade deficit or the NFA position. Thus, the parameters for the restricted dynamic model are the same as the ones for the baseline in Section 3.

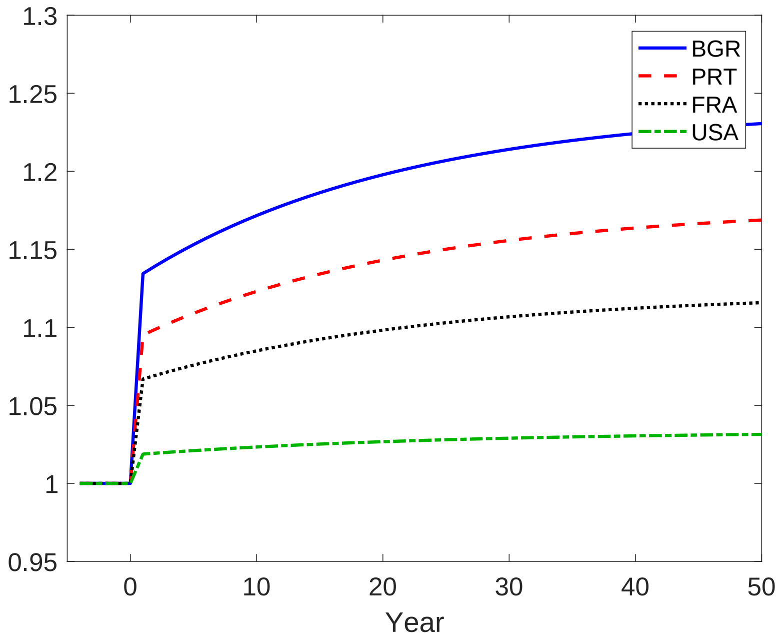
We conduct a 20 percent permanent, uniform, unanticipated trade liberalization in the restricted dynamic model. Similar to the previous subsection, we use the counterfactual transition path for income per worker in the restricted dynamic model (Figure 11) to compute the immediate gain as in equation (11). It turns out that the immediate gain in this restricted dynamic model is practically identical to the immediate gain in the baseline model in section 5.1.

We first compare such immediate gains to the gains across steady states in the restricted dynamic model and quantify the role of capital. We then compare the immediate cost of autarky to the steady-state cost of autarky.

Immediate versus steady-state gains Figure 12 illustrates the ratio of steady-state gains in the restricted dynamic model to the immediate gains measured using (11). The steady-state gains are, on average, 80 percent larger than the immediate gains. The ratio of gains ranges from 1.40 to 2.36. (The ratio is not negative for any country since the initial NFA position for every country is zero.)

The difference between immediate gain and steady-state gain in the restricted dynamic model is driven by: (i) change in TFP and (ii) change in capital in the restricted dynamic

Figure 11: Transition path for income per worker in restricted dynamic model



Notes: Transitions following a uniform, unanticipated, permanent, 20 percent trade liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1. Trade is balanced in each country in every period.

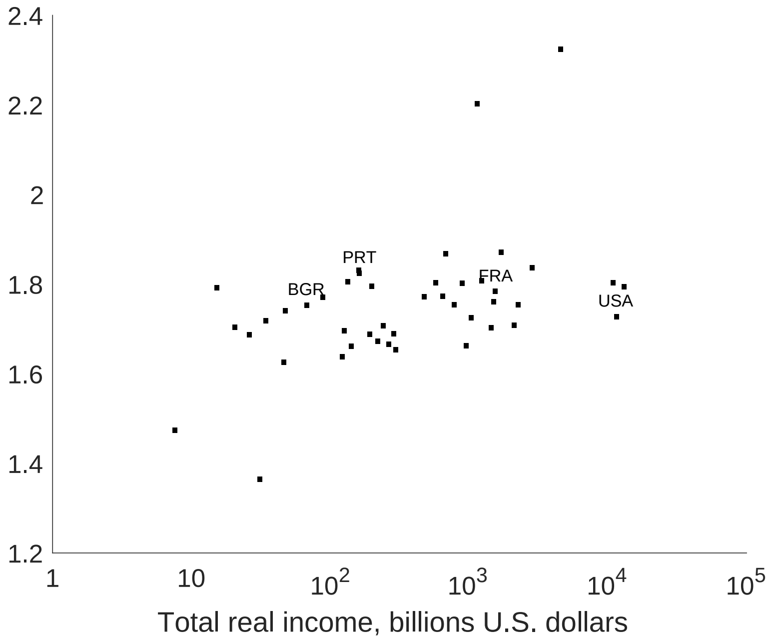
model across steady states. Recall that after the trade liberalization the change in home trade share and, hence, the change in TFP is immediate. Hence, the immediate gain includes (i) but not (ii). To assess the quantitative role of capital accumulation, the *steady-state* income per worker in the restricted dynamic model can be written as

$$y_i \propto \underbrace{\left(\frac{A_{ci}}{B_{ci}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{1-\nu_{ci}}{\theta\nu_{mi}}}}_{\text{TFP}}\left(\underbrace{\left(\frac{A_{xi}}{B_{xi}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{(1-\nu_{xi})}{(1-\alpha)\nu_{mi}}}}_{\text{Capital}}\right)^{\alpha}. \tag{13}$$

We can use equation (13) to decompose the contributions to steady-state gains in each country into a TFP component and a capital-labor ratio component:

$$\Delta\ln(y_i) = -\frac{1}{\theta\nu_{mi}}(1-\nu_{ci})\Delta\ln(\pi_{ii}) - \frac{1}{\theta\nu_{mi}}\left(\frac{\alpha(1-\nu_{xi})}{1-\alpha}\right)\Delta\ln(\pi_{ii}), \tag{14}$$

Figure 12: Ratio of steady-state gains to immediate gains in the restricted dynamic model



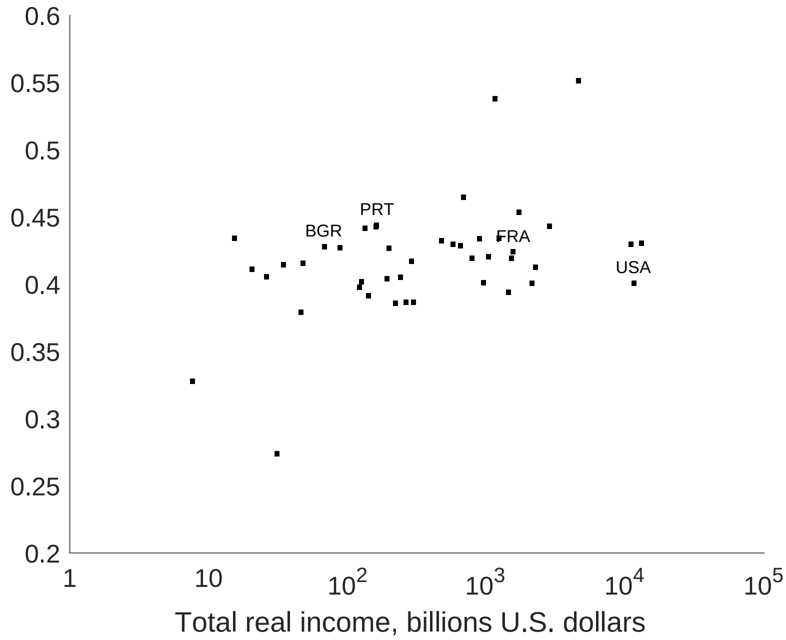
Notes: Welfare gains are computed following a uniform, unanticipated, permanent, 20 percent trade liberalization. Horizontal axis–Total real GDP data for 2014. Vertical axis–Ratio of steady-state gains to immediate gains using the counterfactual income path in the restricted dynamic model. Trade is balanced in each country in every period.

where Δx denotes the difference in x—counterfactual steady-state x minus the initial steady state x. In our counterfactual, the reduction in trade costs would imply a lower home trade share and, hence, a higher income per worker. The steady-state gain (or the change in income per worker) due to the change in capital-labor ratio is the second part of the sum on the right hand side of (14).

The increase in capital-labor ratio in the restricted dynamic model accounts for roughly 43 percent of the gains, on average; the contribution in our sample of countries ranges from 29 percent to 56 percent. See Figure 13. The channel for the increase in the restricted dynamic model is the same as in the baseline model: The increase in TFP and the decrease in relative price of investment induce an increase in the investment rate.

Welfare costs of autarky Equation (14) also helps us compute the steady-state welfare cost of autarky in the restricted dynamic model. Autarky implies home trade share increases to 1 in every country. So, the income decreases and the change in income can be

Figure 13: Capital's contribution to steady-state gains in the restricted dynamic model



Notes: Welfare gains are computed following a uniform, unanticipated, permanent, 20 percent trade liberalization. Horizontal axis–Total real GDP data for 2014. Vertical axis–Contribution of capital to steady-state gains in the restricted dynamic model. Trade is balanced in each country in every period.

written as

$$\Delta \ln(y_i) = \frac{1}{\theta \nu_{mi}}(1 - \nu_{ci}) \ln(\pi_{ii}) + \frac{1}{\theta \nu_{mi}} \left(\frac{\alpha(1 - \nu_{xi})}{1 - \alpha} \right) \ln(\pi_{ii}). \tag{15}$$

The corresponding immediate cost of autarky is

$$\Delta \ln(y_i) = \frac{1}{\theta \nu_{mi}}(1 - \nu_{ci}) \ln(\pi_{ii}). \tag{16}$$

Equation (16) is similar to the *sufficient statistics* formula used by ACR; equation (15) has an additional term that accounts for the change in capital. As in ACR, there is no need to solve the model for the counterfactual home trade share and the observed home trade share is sufficient to compute the welfare costs of autarky. The observed home trade share for Bulgaria is around 0.5, whereas the home trade share for the U.S. is 0.81. The corresponding static costs of autarky are almost 25 percent and 3.4 percent, and the steady-state costs of autarky in the restricted dynamic model are 41 percent and 5.8 percent. Smaller countries in our sample have a higher welfare cost of autarky relative to larger countries.

5.3 A static model

In the third approach, we construct a static model that is essentially the one in Waugh (2010): Capital is an exogenous endowment in each country, there is no investment goods technology (no capital accumulation or adjustment costs), and trade is balanced. The tradable intermediates are used only in the production of final goods and other intermediates. The only difference relative to Waugh (2010) is that the value-added shares in final goods production and intermediate goods production are country-specific. The third approach requires recalibrating the model parameters.

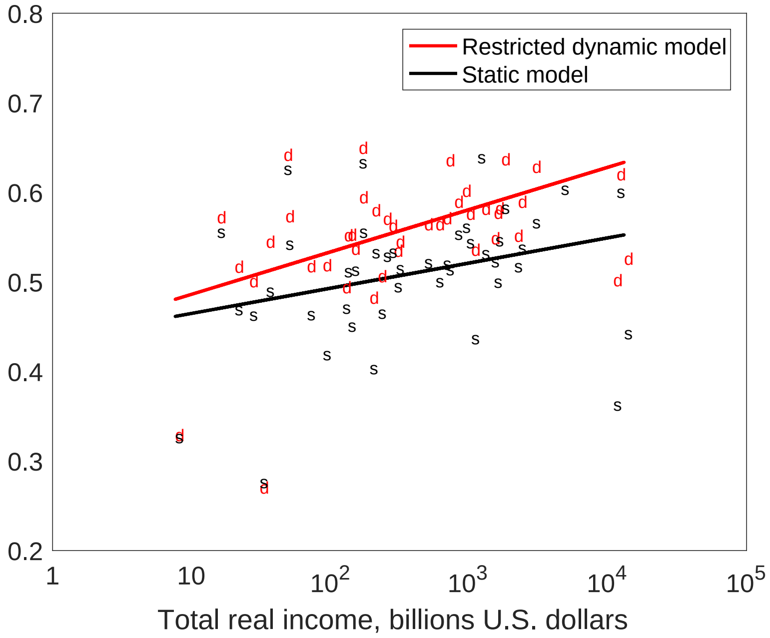
In calibrating the static model, we need to take a position on how we map the static model to the data since capital stock is fixed and does not depend on tradables. We combine consumption and investment goods sectors and interpret the combination as one final good sector. That is, ν_{ci} is the ratio of sum of value added of consumption and investment goods to the sum of gross output of consumption and investment goods, in country i. Figure 14 illustrates ν_{ci} for the static model and for the baseline calibration in Section 3. Recall that ν_{ci} in the restricted dynamic model is the same as in the baseline calibration (Section 3) and is higher for practically every country in our sample, i.e., tradables-intensity in consumption goods is higher in the static model relative to the baseline model.

We then calibrate productivities and trade costs to match income per worker, the price of intermediates relative to consumption, and trade shares, as in Section 3. Note that the trade costs in the static model are the same as in our baseline model since the structural equation used to calibrate the trade costs in the static model is also equation (6) and the data are the same. Finally, the initial capital stock is taken directly from the data, as in the baseline calibration.

We conduct a 20 percent permanent, uniform, unanticipated trade liberalization in the static model. The static gain is different from the immediate gains in the previous subsections since we compute it from a stand-alone static model with its own parameters.

Several caveats are in order here. First, the static gains accrue immediately after the liberalization and there is no cost to increasing consumption. Second, the structural parameters of the static world are different from those in the dynamic world. In the static world none of the tradables are allocated to inputs that enhance future production possibilities.

Figure 14: Value-added share in consumption goods sector: ν_c



Notes: The letters s and d in each scatter plot denote the value added share in the final goods sector in the static model and the value added share in the consumption goods sector in the baseline model, respectively. Horizontal axis–Total real GDP data for 2014. The value of ν_c in the restricted dynamic model is the same as that in Figure 1.

To begin, recall that the income per worker in the static model is given by

$$y_i \propto \underbrace{\left(\frac{A_{ci}}{B_{ci}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{1-\nu^s_{ci}}{\theta\nu_{mi}}}}_{\text{TFP}}(k_i)^{\alpha}, \tag{17}$$

where the superscript "s" on ν_{ci} denotes the static value. The static gain is computed according to

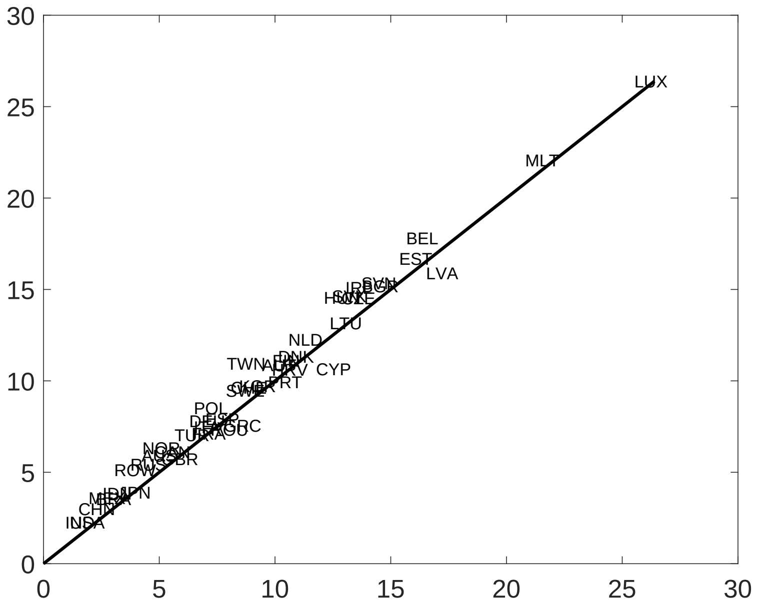
$$1+\frac{\lambda_i^{static}}{100}=\frac{\hat{y}_i}{y_i^{\star}}, \tag{18}$$

where $\hat{y}_i$ is the income per worker in country i after the trade liberalization.

The immediate gain computed using (11) in Section 5.1 and the static gain computed using (18) are not quantitatively different. Figure 15 illustrates the two gains.

Despite the fact that the structural parameter in the final goods sector in the static model

Figure 15: Immediate gains in the baseline model and gains in the static model



Notes: Gains following a uniform, unanticipated, permanent, 20 percent trade liberalization. Horizontal axis–Immediate change in income per worker along the transition path in the baseline dynamic model. Vertical axis–Gain in the static model. The solid line is the 45-degree line.

in this section is different from the one used in Section 5.1 and the fact that the immediate gain computed in Section 5.1 used just a component of the transition path, the two gains look the same. Thus, Figure 15 implies that the role of capital accumulation quantified in Section 5.1 and Section 5.2 continues to hold. As a result, the dynamic gain in our baseline would be, on average, 35 percent larger than the static gain; the steady-state gain in the restricted dynamic model would be, on average, 80 percent larger than the static gain.

Summary The immediate gains in our baseline model are practically identical to the gains in a static model where the calibrated intensity of tradables in final goods production is higher. The dynamic gains including transition in our model are 35 percent higher than the static gains. The steady-state gains in a balanced-trade version of our model are 80 percent higher than the static gains and capital contributes to 43 percent of the gains across steady states.

6 Roles of trade imbalance and intensity of tradables

In this section, we examine the quantitative role of two features of our baseline model in Section 2: trade imbalances and intensity of tradables. In contrast to our model with endogenous trade imbalances, the assumption of balanced trade limits the extent of consumption smoothing over time and, hence, the dynamic gains. In Section 6.1, we quantify the role of trade imbalances in our baseline model by conducting a few counterfactual experiments in models with balanced trade. In our baseline model, the tradable intensities $1 - \nu_c$ and $1 - \nu_x$ directly affect income at each point in time. Furthermore, the difference $\nu_x - \nu_c$ affects how the relative price of investment responds to a trade liberalization, which in turn affects the response of the investment rate. In Section 6.2, we examine the quantitative role of the absolute value of the tradable intensities and the role of the difference in tradable intensities in each country.

6.1 Balanced trade versus trade imbalances

In this subsection, we conduct two comparisons. First, we construct a variant of our baseline model where we impose balanced trade in each country in the initial steady state, but allow for trade imbalances along the counterfactual transition path after the trade liberalization. We recalibrate the model and compare the welfare gains in this model to one where trade is balanced in each country in the initial steady state and in each period after the liberalization. Second, we compare the gains in our baseline model (Section 4) to the variant above: a model with balanced trade in the initial steady state, but not along the counterfactual transition.

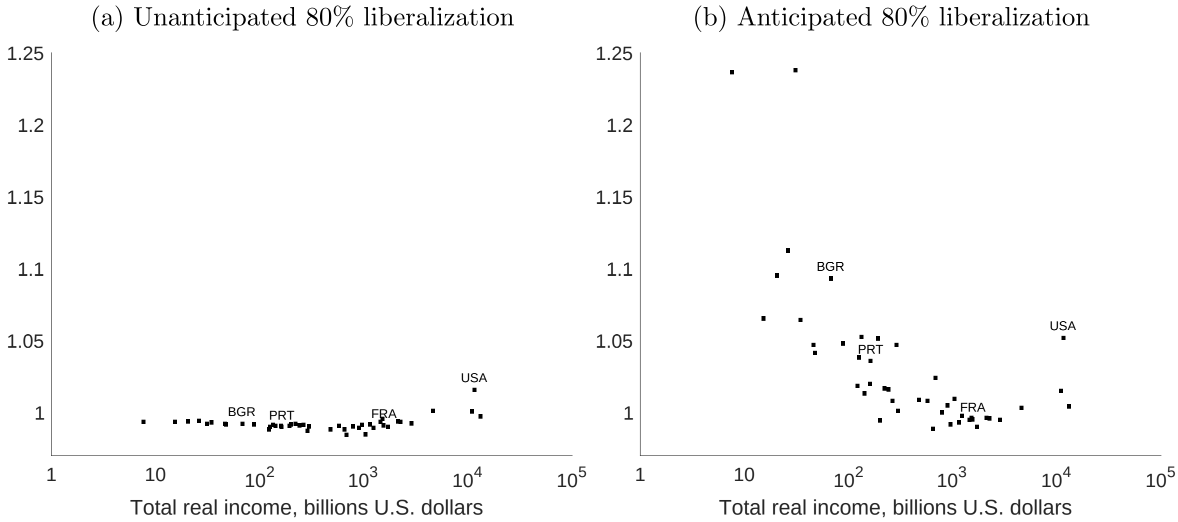
Comparison 1 Recalibrating the variant of our baseline model does not change the structural parameters, productivities, or trade costs. It changes the initial NFA position: In our baseline model, the initial NFA position was consistent with the observed trade imbalance, but in the variant, trade is balanced and the initial NFA position is zero.

In the initial steady state, MPKs are equalized across countries in the variant of our baseline model and in the model with balanced trade in each period after the liberalization. However, in the latter MPKs differ across countries along the transition, while in the model with trade imbalances MPKs are equalized because of bond trade across countries. The cross-country trade in assets implies that resources are reallocated more efficiently across countries, which helps smooth consumption over time in each country.

We begin with the same unanticipated, permanent, and uniform trade liberalization of

20 percent. We find that the gains from trade are virtually identical in the two models. Furthermore, the gains are almost identical in the two models even for large unanticipated, permanent, and uniform trade liberalizations. Figure 16, panel a, illustrates the ratio of gains in the model with trade imbalances along the transition to the gains in the model with balanced trade for a trade liberalization of 80 percent.

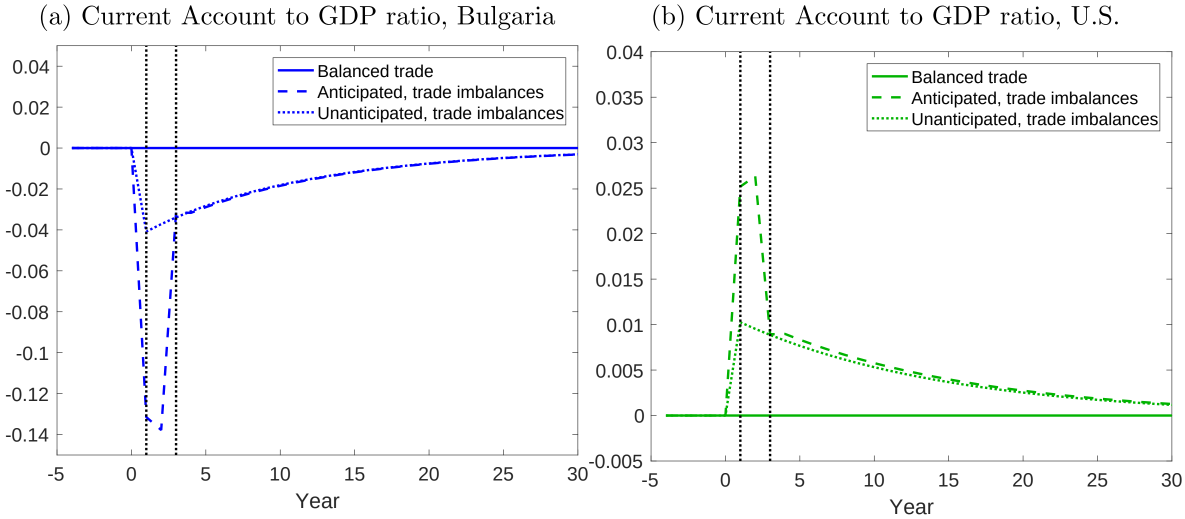
Figure 16: Ratio of gains: Trade imbalances along the transition to balanced trade



Notes: Welfare gains are computed following a uniform, permanent, 80 percent trade liberalization. In the unanticipated case, the liberalization occurs in period 1. In the anticipated case, the liberalization is announced in period 1 but occurs in period 3. Horizontal axis–Total real GDP data for 2014. Vertical axis–Ratio of gains in the model with trade imbalances along the transition to the gains in the model with balanced trade in every period.

It turns out that *anticipation* of a trade liberalization yields different implications for welfare gains. Suppose that in period 1 there is an announcement that the trade liberalization will take effect in period 3. For the case of anticipated, permanent, and uniform 80 percent trade liberalization in both models, Figure 16, panel b illustrates that the model with endogenous trade imbalances yields higher gains relative to the model with balanced trade. In Bulgaria, for instance, the gain is 10 percent higher when the liberalization is anticipated. After trade liberalization, resources flow from the United States to Bulgaria in the model with endogenous trade imbalances, resulting in a current account deficit in Bulgaria and a current account surplus the United States. See Figure 17. These patterns imply that, relative to a model with balanced trade, Bulgaria front-loads consumption and the United States back-loads consumption. In the case of an anticipated trade liberalization, these effects will materialize before the liberalization happens, amplifying the effects of the trade liberalization on consumption. The United States decreases consumption at the time of the announcement, by running a large current account surplus. This feature is absent in a model with balanced trade and in the model with unanticipated trade liberalization. By the time the liberalization occurs in period 3, each country's current account is approximately at the same level as it would have been in an unanticipated trade liberalization.

Figure 17: Current account in unanticipated and anticipated trade liberalizations

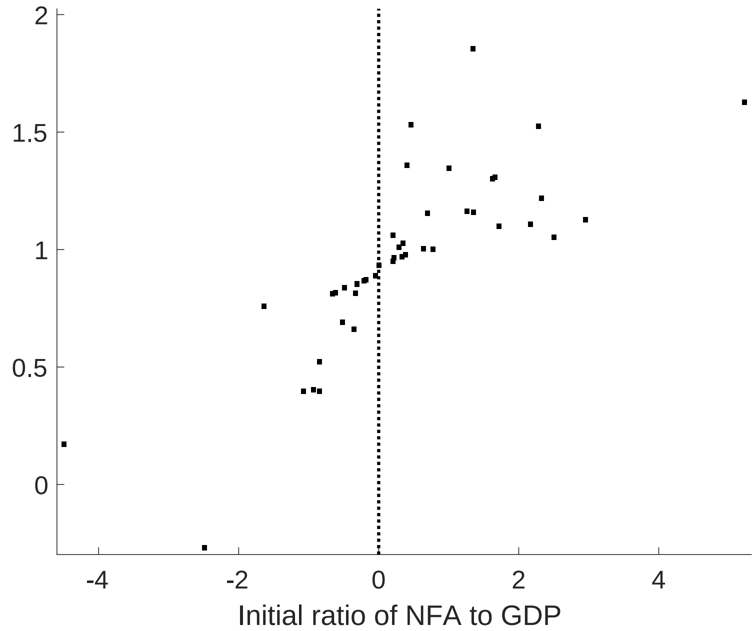


Notes: Transitions following a uniform, permanent 20 percent liberalization. In the unanticipated case, the liberalization occurs in period 1. In the anticipated case, the liberalization is announced in period 1 but occurs in period 3.

Comparison 2 We compare the welfare gains in our baseline model to the gains in the variant of the baseline model from an unanticipated, permanent, and uniform trade liberalization of 20 percent. The key difference between the two models is in the initial steady state: In our baseline the current account balance is zero in each country and the NFA position is different across countries, whereas in the variant of the baseline the NFA position is zero and trade is balanced in each country. Figure 18 illustrates the ratio of gains in our baseline model to the gains in the variant.

The gains in the variant do not depend on the initial NFA position since it is zero in all countries, but the gains in our baseline do depend on the initial NFA position. Countries that start with a negative NFA position in the baseline lose relative to the variant where the

Figure 18: Ratio of gains with initial trade imbalances to gains with initial balanced trade



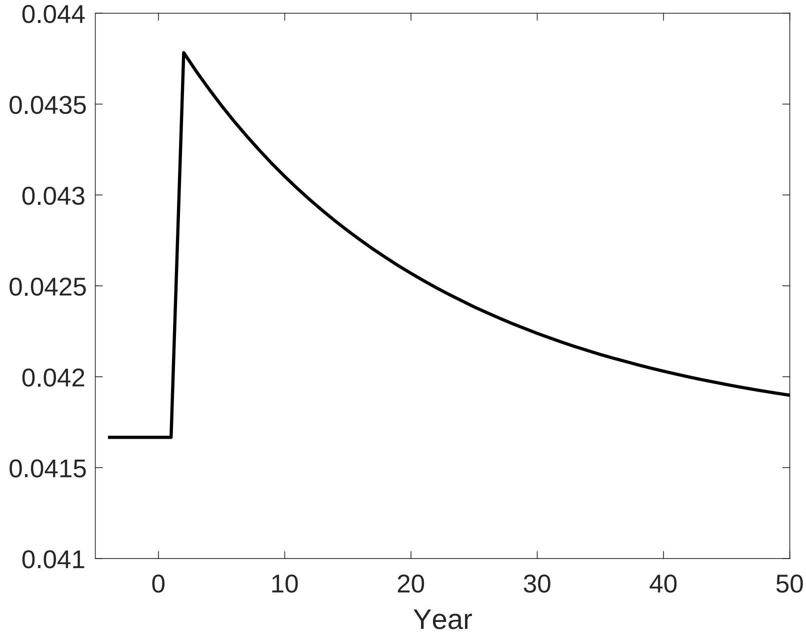
Notes: Welfare gains are computed following a uniform, unanticipated, permanent, 20 percent trade liberalization. Horizontal axis–Ratio of net-foreign assets to GDP in the initial steady state. Vertical axis–Ratio of gains in a model with trade imbalances in the initial steady state relative to those in a model with balanced trade in the initial steady state. In both models trade imbalances are endogenous during the counterfactual transition path after liberalization.

initial NFA position is zero. Along the same lines, countries that start with a positive NFA position in the baseline gain relative to the variant. This is because in both models, trade liberalization increases the world interest rate on impact. Figure 19 illustrates the transition path of the world interest rate. The increase in interest rates implies that countries with initial debt suffer and countries with initial positive assets benefit.

The ratio of gains in a model with trade imbalances in the initial steady state relative to those in a model with balanced trade in the initial steady state is increasing with initial steady-state NFA position. Countries that start with a negative NFA position, have a ratio lower than one, while countries that start with positive NFA have a ratio larger than one. A case in point is Norway, which has a negative ratio in Figure 18. In our initial steady state, Norway has a large negative NFA and large net exports in 2014 due to high oil prices. The increase in the interest rate severely affects Norway's gains.[5]

[5] See Gourinchas and Jeanne (2006) who find that the gains from a financial liberalization are small compared to the gains from a trade liberalization.

Figure 19: World interest rate



Notes: Transition following a uniform, unanticipated, permanent, 20 percent trade liberalization. The liberalization occurs in period 1. Vertical axis–Ratio of gains in a model with trade imbalances in the initial steady state relative to those in a model with balanced trade in the initial steady state. In both models trade imbalances are endogenous during the counterfactual transition path after liberalization.

6.2 Intensity of tradables in consumption and investment goods

Recall that in our baseline dynamic model $1 - \nu_{ci}$ denotes the tradables-intensity for consumption goods and $1 - \nu_{xi}$ denotes that for investment goods in country i. These are heterogeneous across countries. The contribution of TFP to the welfare gains depends on the value of $1-\nu_c$, whereas the contribution of capital in the baseline model depends not only on the value of $1 - \nu_x$ but also on the difference $\nu_x - \nu_c$ in each country, since the difference affects how the relative price of investment responds to a trade liberalization, which in turn affects the response of the investment rate. In this subsection, we examine (i) the implications of $\nu_{xi} - \nu_{ci}$ for the response of the relative price of investment and (ii) the implications of ν_{ci} and ν_{xi} for the responses of TFP and capital accumulation, respectively.

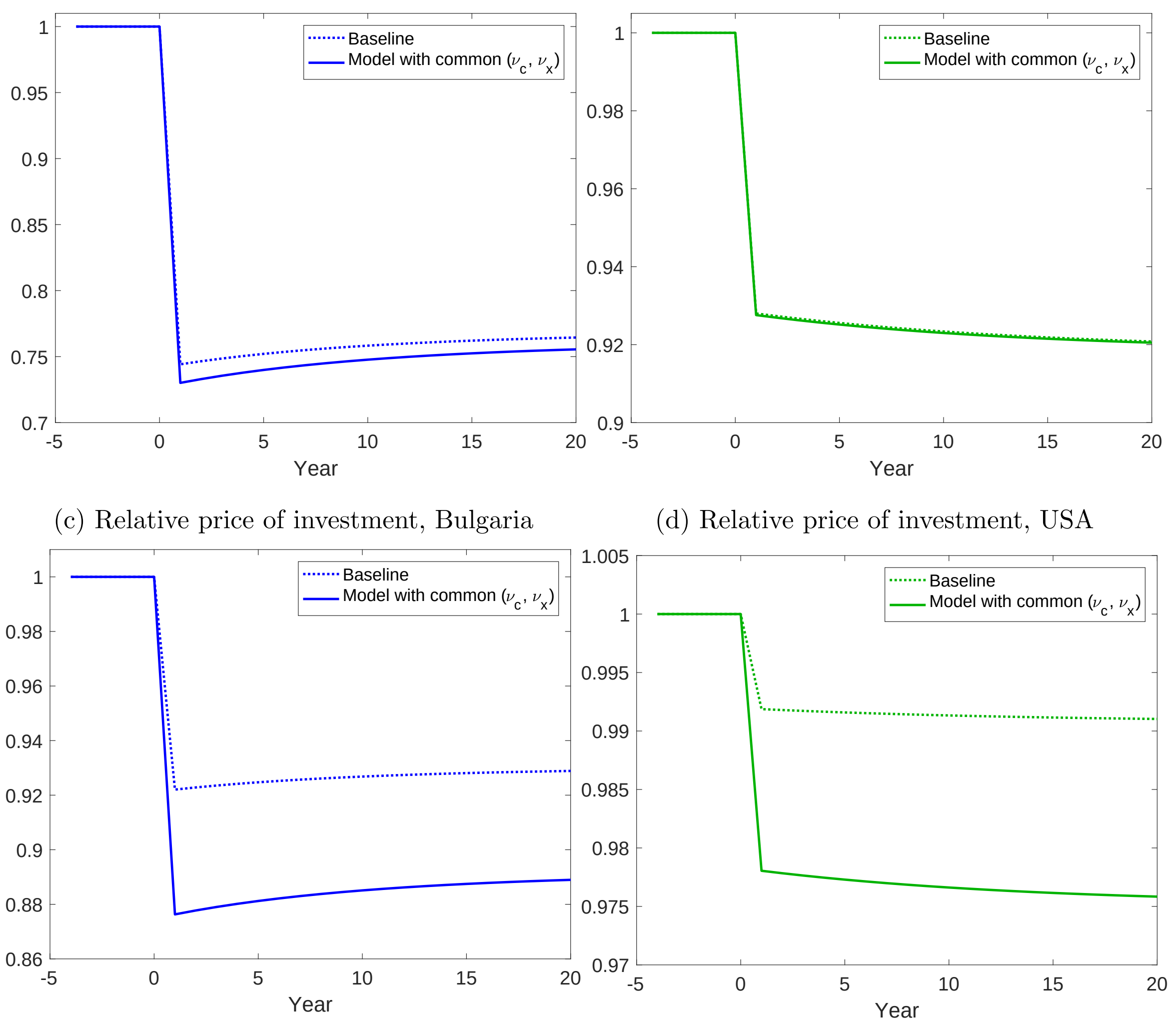
Effect on the relative price of investment In our baseline calibration in Section 3, investment goods are more tradable intensive than consumption goods ($\nu_{xi} < \nu_{ci}$), which implies that the relative price of investment falls after a trade liberalization. The magnitude of the decline in relative price of investment differs across countries and affects the investment rate and the capital-labor ratio. Equation (3) shows that the change in the relative price of investment is driven by: (i) the change in the home trade share and (ii) $\nu_{xi} - \nu_{ci}$.

In order to isolate the roles of (i) and (ii), we first show that the response of the home trade share is virtually independent of the values of ν_{xi} and ν_{ci}. To illustrate this, suppose that all countries have the same ν_{xi} and the same ν_{ci}. Specifically, we find the country with the maximum difference between ν_x and ν_c, which in our model is India. Denote India's tradables-intensities by $\bar{\nu}_x$ and $\bar{\nu}_c$. We set $\nu_{xi} = \bar{\nu}_x$ and $\nu_{ci} = \bar{\nu}_c$ for all i; this preserves the difference between ν_x and ν_c but eliminates the heterogeneity across countries in the tradable intensities. We then implement the counterfactual 20 percent trade liberalization. Figure 20 illustrates the transition path for home trade share and relative price of investment for Bulgaria and United States. We find that the path for home trade share after the liberalization is almost identical to the path in our baseline model with heterogeneous tradable intensities. However, the relative price falls by more than in the baseline model because of the greater difference between the tradables intensities in consumption goods and investment goods.

Effects on TFP and capital accumulation Next, we explore how the magnitudes of ν_{xi} and ν_{ci} individually affect the response of TFP and capital accumulation after a trade liberalization. Since changes in these parameters have an effect on the relative price of investment, we consider two alternative specifications: (i) We keep ν_{ci} fixed to its calibrated value and increase ν_{xi} to equal ν_{ci}, thereby making investment goods less tradable intensive, and (ii) we keep ν_{xi} fixed to its calibrated value and decrease ν_{ci} to equal ν_{xi}, thereby making consumption goods more tradable intensive. Note that both (i) and (ii) allow for ν_{xi} and ν_{ci} to vary across countries, but ensure that the relative price does not respond to a trade liberalization since $\nu_{xi} - \nu_{ci} = 0$. In both specifications, we again consider the 20 percent trade liberalization and examine the response of TFP and capital accumulation.

Figure 21 illustrates the results for Bulgaria and the United States. In specification (i), when we fix ν_{ci} to its calibrated value, TFP follows the same path as in the baseline model, even though ν_{xi} differs from its calibrated value. In (ii), when we fix ν_{xi} to its calibrated value and increase ν_{ci}, TFP is higher at every point in time. By making consumption goods more tradables-intensive in (ii), the production possibility frontier shifts more in response to reductions in trade frictions (see Figures 21a and 21b). Note that the change in TFP depends on the change in home trade share, π_{ii}, and the value of ν_{ci}. However, the change in the

Figure 20: Transitions with no cross-country heterogeneity in tradable intensities



(a) Home trade share, Bulgaria

(b) Home trade share, USA

(c) Relative price of investment, Bulgaria

(d) Relative price of investment, USA

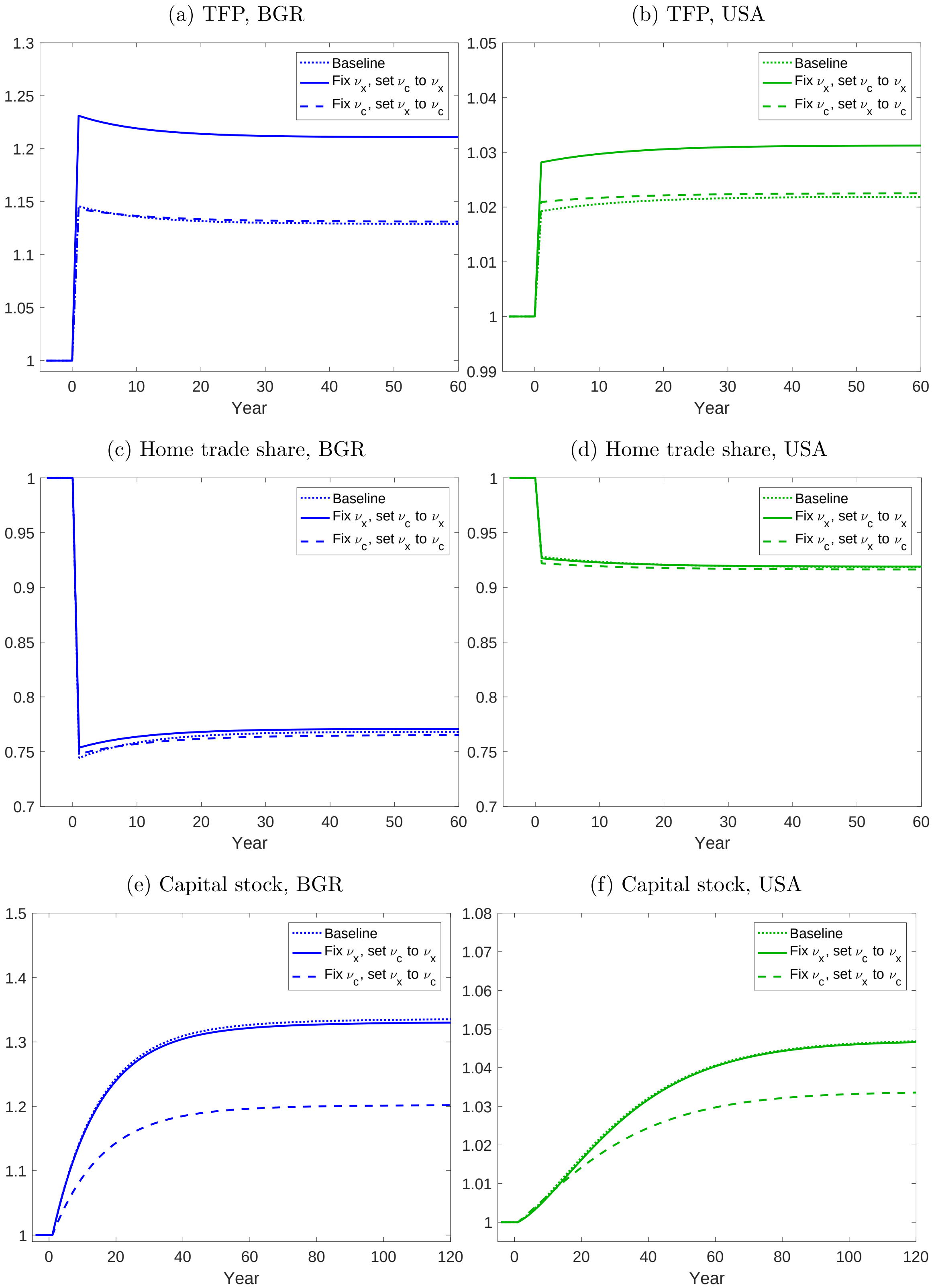
Notes: Transitions following a uniform, unanticipated, permanent, 20 percent liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1. Model with common (ν_c, ν_x) preserves the difference between ν_c and ν_x but eliminates the heterogeneity across countries in ν_c and in ν_x.

home trade share is virtually invariant to the values of ν_{ci} and ν_{xi} (see Figures 21c and 21d). Therefore, the difference in the paths for TFP in country i between the two specifications is determined entirely by the value of ν_{ci}.

Similarly, the difference in the paths for the capital stock across the two counterfactuals is determined by the value of ν_{xi}. When we fix ν_{xi} to its calibrated value, capital follows the same path as in the baseline model, even though ν_{ci} differs from its calibrated value. Instead, when we increase ν_{xi} to the fixed value of ν_{ci}, capital is lower at every point in time (see Figures 21e and 21f) since the investment goods production is less tradables-intensive in specification (i).

In sum, the value of the tradable-intensities in investment goods production determines the transition path for capital, while the value of the tradable-intensities in consumption goods production determines the transition path for TFP.

Figure 21: Transitions with equal tradable intensities in consumption and investment



Notes: Transitions following a uniform, unanticipated, permanent, 20 percent liberalization. Initial steady state is normalized to 1. The liberalization occurs in period 1. One specification keeps ν_{ci} fixed to its calibrated value and increases ν_{xi} to equal ν_{ci}. The other specification keeps ν_{xi} fixed to its calibrated value and decreases ν_{ci} to equal ν_{xi}. In both specifications, the relative price of investment does not respond to a trade liberalization since $\nu_{xi} - \nu_{ci} = 0$.

7 Conclusion

We build a multicountry trade model with capital accumulation to study dynamic welfare gains. In our model, tradable intermediates are used in the production of final consumption goods and investment goods with different intensities. Cross-country asset trades generate endogenous trade imbalances and help smooth consumption over time.

Trade liberalization reduces the relative price of investment, allowing countries to invest more and therefore attain permanently higher capital-labor ratios. Trade liberalization also increases total factor productivity, which increases the rate of return to investment and, hence, the investment rate. As capital accumulates, consumption increases and the welfare gains accrue over time.

For an unanticipated and uniform reduction in trade costs, we find that the gains are negatively correlated with size, financial resources flow from larger countries to smaller countries, countries with larger short-run trade deficits accumulate capital faster, smaller countries front-load their consumption while larger countries do the opposite, the gains are nonlinear in the reduction in trade costs, and capital accumulation accounts for substantial gains relative to a model where capital is fixed.

The net foreign asset position before the liberalization and the intensities of tradables in investment goods production and consumption goods production are quantitatively important for the gains. The liberalization increases the world interest rate on impact, which implies that countries with initial debt suffer and countries with initial positive assets benefit. The transition path for capital and TFP depend almost entirely on the tradable-intensities of the investment and consumption goods production, respectively.

Our computational algorithm efficiently solves for the exact transitional dynamics for a system of second-order, nonlinear difference equations. Our method iterates on prices using excess demand functions and does not involve costly gradient calculations, and delivers the transition paths for all countries in less than two hours. Thus, our method is useful for solving multicountry trade models with a large state space. Our solution method can also be used to analyze other types of changes in trade costs such as multilateral trade agreements with gradual reductions in trade barriers (e.g., European Union and NAFTA), anticipated changes in trade costs (e.g., Brexit) and other types of models with multiple sectors and input-output linkages.

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Appendix

A Equilibrium conditions

We describe each equilibrium condition in detail below.

Household optimization The representative household chooses a path for consumption that satisfies two intertemporal Euler equations associated with the one-period bond and capital:

$$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{1+q_{t+1}}{P_{cit+1}/P_{cit}}\right)^{\sigma}$$

and

$$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{ixt+1}} - \Phi_2(K_{it+2}, K_{it+1})}{\Phi_1(K_{it+1}, K_{it})}\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma},$$

where $\Phi_1(\cdot,\cdot)$ and $\Phi_2(\cdot,\cdot)$ denote the first derivatives of the adjustment-cost function with respect to the first and second arguments, respectively:

$$\Phi_1(K', K) = \left(\frac{1}{\chi}\right)^{\frac{1}{\lambda}}\left(\frac{1}{\lambda}\right)\left(\frac{K'}{K} - (1-\delta)\right)^{\frac{1-\lambda}{\lambda}}$$
$$\Phi_2(K', K) = \left(\frac{1}{\chi}\right)^{\frac{1}{\lambda}}\left(\frac{1}{\lambda}\right)\left(\frac{K'}{K} - (1-\delta)\right)^{\frac{1-\lambda}{\lambda}}\left((\lambda-1)\frac{K'}{K} - \lambda(1-\delta)\right).$$

Combining the household's budget constraint and the capital accumulation technology and rearranging, we get:

$$P_{cit}C_{it} + P_{xit}\Phi(K_{it+1}, K_{it}) + \mathcal{A}_{it+1} = r_{it}K_{it} + w_{it}L_i + q_t\mathcal{A}_{it}.$$

Firm optimization Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety v, produced in country j and purchased by country i, as $p_{mij}(v)$. Then $p_{mij}(v) = p_{mjj}(v)d_{ij}$; in country j, $p_{mjj}(v)$ is also the marginal cost of producing variety v. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_{mi}(v) = \min_{j=1,\ldots,I}[p_{mjj}(v)d_{mij}]$.

The price of the composite good in country i at time t is then

$$P_{mit} = \gamma \left[\sum_{j=1}^{I} (u_{jt} d_{ij})^{-\theta} T_{mj} \right]^{-\frac{1}{\theta}},$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_{mj}}\right)^{\alpha\nu_{mj}} \left(\frac{w_{jt}}{(1-\alpha)\nu_{mj}}\right)^{(1-\alpha)\nu_{mj}} \left(\frac{P_{jt}}{1-\nu_{mj}}\right)^{1-\nu_{mj}}$ is the unit cost for a bundle of inputs for intermediate goods producers in country n at time t.

Next we define total factor usage in the intermediates sector by aggregating across the individual varieties.

$$K_{mit} = \int_0^1 K_{mit}(v)dv, \qquad L_{mit} = \int_0^1 L_{mit}(v)dv,$$
$$M_{mit} = \int_0^1 M_{mit}(v)dv, \qquad Y_{mit} = \int_0^1 Y_{mit}(v)dv.$$

The term $L_{mit}(v)$ denotes the labor used in the production of variety v at time t. If country i imports variety v at time t, then $L_{mit}(v) = 0$. Hence, L_{mit} is the total labor used in sector m in country i at time t. Similarly, K_{mit} is the total capital used, M_{mit} is the total intermediates used as an input, and Y_{mit} is the total output of intermediates.

Cost minimization by firms implies that, within each sector $b \in \{c, m, x\}$, factor expenses exhaust the value of output:

$$r_{it} K_{bit} = \alpha\nu_{bi} P_{bit} Y_{bit},$$
$$w_{it} L_{bit} = (1-\alpha)\nu_{bi} P_{bit} Y_{bit},$$
$$P_{mit} M_{bit} = (1-\nu_{bi}) P_{bit} Y_{bit}.$$

That is, the fraction $\alpha\nu_{bi}$ of the value of each sector's production compensates capital services, the fraction $(1-\alpha)\nu_{bi}$ compensates labor services, and the fraction $1-\nu_{bi}$ covers the cost of intermediate inputs; there are zero profits.

Trade flows The fraction of country i's expenditures allocated to intermediate varieties produced by country j is given by

$$\pi_{ijt} = \frac{(u_{mjt} d_{ijt})^{-\theta} T_{mj}}{\sum_{j=1}^{I} (u_{mjt} d_{ij})^{-\theta} T_{mj}},$$

where u_{mjt} is the unit cost of intermediate varieties in country j.

Market clearing conditions The domestic factor market clearing conditions are:

$$\sum_{b\in\{c,m,x\}} K_{bit} = K_{it}, \qquad \sum_{b\in\{c,m,x\}} L_{bit} = L_i, \qquad \sum_{b\in\{c,m,x\}} M_{bit} = M_{it}.$$

The first two conditions impose that the capital and labor markets clear in country i at each time t. The third condition requires that the use of the composite good equals its supply. Its use consists of demand by firms in each sector. Its supply consists of both domestically and foreign-produced varieties.

The next set of conditions require that goods markets clear.

$$C_{it} = Y_{cit}, \qquad X_{it} = Y_{xit}, \qquad \sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}.$$

The first condition states that the quantity of (nontradable) consumption demanded by the representative household in country i must equal the quantity produced by country i. The second condition says the same for the investment good. The third condition imposes that the value of intermediates produced by country i has to be absorbed globally. Recall that $P_{mjt}M_{bjt}$ is the value of intermediate inputs that country i uses in production in sector b. The term π_{jit} is the fraction of country j's intermediate good expenditures sourced from country i. Therefore, $P_{mjt}M_{bjt}\pi_{jit}$ denotes the value of trade flows from country i to j.

Finally, we impose an aggregate resource constraint in each country: Net exports equal zero. Equivalently, gross output equals gross absorption:

$$B_{it} = P_{mit}Y_{mit} - P_{mit}M_{it} + q_t\mathcal{A}_{it}.$$

Given an initial NFA position and capital stock, the equilibrium transition path consists of the following objects: $\{\vec{w}_t\}_{t=1}^{T}$, $\{\vec{r}_t\}_{t=1}^{T}$, $\{q_t\}_{t=1}^{T}$, $\{\vec{P}_{ct}\}_{t=1}^{T}$, $\{\vec{P}_{mt}\}_{t=1}^{T}$, $\{\vec{P}_{xt}\}_{t=1}^{T}$, $\{\vec{C}_t\}_{t=1}^{T}$, $\{\vec{X}_t\}_{t=1}^{T}$, $\{\vec{K}_t\}_{t=1}^{T+1}$, $\{\vec{B}_t\}_{t=1}^{T}$,$\{\vec{\mathcal{A}}_t\}_{t=1}^{T+1}$, $\{\vec{Y}_{ct}\}_{t=1}^{T}$, $\{\vec{Y}_{mt}\}_{t=1}^{T}$, $\{\vec{Y}_{xt}\}_{t=1}^{T}$, $\{\vec{K}_{ct}\}_{t=1}^{T}$, $\{\vec{K}_{mt}\}_{t=1}^{T}$, $\{\vec{K}_{xt}\}_{t=1}^{T}$, $\{\vec{L}_{ct}\}_{t=1}^{T}$, $\{\vec{L}_{mt}\}_{t=1}^{T}$, $\{\vec{L}_{xt}\}_{t=1}^{T}$, $\{\vec{M}_{ct}\}_{t=1}^{T}$, $\{\vec{M}_{mt}\}_{t=1}^{T}$, $\{\vec{M}_{xt}\}_{t=1}^{T}$, $\{\vec{\vec{\pi}}_t\}_{t=1}^{T}$ (The double-arrow notation on $\vec{\vec{\pi}}_t$ is used to indicate that this is an $I \times I$ matrix in each period t). Table A.1 provides a list of equilibrium conditions that these objects must satisfy.

In this environment, the world interest rate is strictly nominal. That is, the prices map into current units, as opposed to constant units. In other words, the model can be rewritten

Table A.1: Dynamic equilibrium conditions in model with trade imbalances

1	$r_{it}K_{cit} = \alpha\nu_{ci}P_{cit}Y_{cit}$	$\forall(i,t)$
2	$r_{it}K_{mit} = \alpha\nu_{mi}P_{mit}Y_{mit}$	$\forall(i,t)$
3	$r_{it}K_{xit} = \alpha\nu_{xi}P_{xit}Y_{xit}$	$\forall(i,t)$
4	$w_{it}L_{cit} = (1-\alpha)\nu_{ci}P_{cit}Y_{cit}$	$\forall(i,t)$
5	$w_{it}L_{mit} = (1-\alpha)\nu_{mi}P_{mit}Y_{mit}$	$\forall(i,t)$
6	$w_{it}L_{xit} = (1-\alpha)\nu_{xi}P_{xit}Y_{xit}$	$\forall(i,t)$
7	$P_{mit}M_{cit} = (1-\nu_{ci})P_{cit}Y_{cit}$	$\forall(i,t)$
8	$P_{mit}M_{mit} = (1-\nu_{mi})P_{mit}Y_{mit}$	$\forall(i,t)$
9	$P_{mit}M_{xit} = (1-\nu_{xi})P_{xit}Y_{xit}$	$\forall(i,t)$
10	$K_{cit} + K_{mit} + K_{xit} = K_{it}$	$\forall(i,t)$
11	$L_{cit} + L_{mit} + L_{xit} = L_{it}$	$\forall(i,t)$
12	$M_{cit} + M_{mit} + M_{xit} = M_{it}$	$\forall(i,t)$
13	$C_{it} = Y_{cit}$	$\forall(i,t)$
14	$\sum_{j=1}^{I} P_{mjt}M_{jt}\pi_{jit} = P_{mit}Y_{mit}$	$\forall(i,t)$
15	$X_{it} = Y_{xit}$	$\forall(i,t)$
16	$P_{cit} = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_{it}}{\alpha\nu_{ci}}\right)^{\alpha\nu_{ci}}\left(\frac{w_{it}}{(1-\alpha)\nu_{ci}}\right)^{(1-\alpha)\nu_{ci}}\left(\frac{P_{mit}}{1-\nu_{ci}}\right)^{1-\nu_{ci}}$	$\forall(i,t)$
17	$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{mjt}d_{ijt})^{-\theta}T_{mjt}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
18	$P_{xit} = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_{it}}{\alpha\nu_{xi}}\right)^{\alpha\nu_{xi}}\left(\frac{w_{it}}{(1-\alpha)\nu_{xi}}\right)^{(1-\alpha)\nu_{xi}}\left(\frac{P_{mit}}{1-\nu_{xi}}\right)^{1-\nu_{xi}}$	$\forall(i,t)$
19	$\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta}T_{mjt}}{\sum_{j=1}^{I}(u_{mjt}d_{ijt})^{-\theta}T_{mjt}}$	$\forall(i,j,t)$
20	$P_{cit}C_{it} + P_{xit}X_{it} + B_{it} = r_{it}K_{it} + w_{it}L_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$
21	$\mathcal{A}_{it+1} = \mathcal{A}_{it} + B_{it}$	$\forall(i,t)$
22	$K_{it+1} = (1-\delta)K_{it} + \chi X_{it}^{\lambda}K_{it}^{1-\lambda}$	$\forall(i,t)$
23	$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{xit+1}} - \Phi_2(K_{it+2},K_{it+1})}{\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}$	$\forall(i,t)$
24	$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{1+q_{t+1}}{P_{cit+1}/P_{cit}}\right)^{\sigma}$	$\forall(i,t)$
25	$B_{it} = P_{mit}Y_{mit} - P_{mit}M_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$

Note: The term $u_{mjt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mjt}}{1-\nu_m}\right)^{1-\nu_m}$.

so that all prices are quoted in time-1 units (like an Arrow-Debreu world) with the world interest rate of zero and the equilibrium would yield identical quantities. Since our choice of numéraire is world GDP in each period, the world interest rate reflects the relative valuation of world GDP at two points in time. This interpretation helps guide the solution procedure.

In general, in models with trade imbalances, the steady state is not independent of the transition path that leads up to that steady state. We treat the initial steady state as independent of the prior transition by fixing the NFA position. With this NFA, all other steady-state equilibrium conditions are pinned down uniquely. The new steady state is determined jointly with the transition path. The solution to the initial steady-state consists of 23 objects: $\vec{w}^\star$, $\vec{r}^\star$, $q^\star$, $\vec{P}_c^\star$, $\vec{P}_m^\star$, $\vec{P}_x^\star$, $\vec{C}^\star$, $\vec{X}^\star$, $\vec{K}^\star$, $\vec{M}^\star$, $\vec{Y}_c^\star$, $\vec{Y}_m^\star$, $\vec{Y}_x^\star$, $\vec{K}_c^\star$, $\vec{K}_m^\star$, $\vec{K}_x^\star$, $\vec{L}_c^\star$, $\vec{L}_m^\star$, $\vec{L}_x^\star$, $\vec{M}_c^\star$, $\vec{M}_m^\star$, $\vec{M}_x^\star$, $\vec{\vec{\pi}}^\star$ (we use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix). Table A.2 provides a list of 24 conditions that these objects must satisfy. One market clearing equation is redundant (condition 12 in our algorithm).

Table A.2: Steady-state conditions in the balanced trade model

1	$r_i^\star K_{ci}^\star = \alpha\nu_{ci} P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
2	$r_i^\star K_{mi}^\star = \alpha\nu_{mi} P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
3	$r_i^\star K_{xi}^\star = \alpha\nu_{xi} P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
4	$w_i^\star L_{ci}^\star = (1-\alpha)\nu_{ci} P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
5	$w_i^\star L_{mi}^\star = (1-\alpha)\nu_{mi} P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
6	$w_i^\star L_{xi}^\star = (1-\alpha)\nu_{xi} P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
7	$P_{mi}^\star M_{ci}^\star = (1-\nu_{ci}) P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
8	$P_{mi}^\star M_{mi}^\star = (1-\nu_{mi}) P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
9	$P_{mi}^\star M_{xi}^\star = (1-\nu_{xi}) P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
10	$K_{ci}^\star + K_{mi}^\star + K_{xi}^\star = K_i^\star$	$\forall(i)$
11	$L_{ci}^\star + L_{mi}^\star + L_{xi}^\star = L_i$	$\forall(i)$
12	$M_{ci}^\star + M_{mi}^\star + M_{xi}^\star = M_i^\star$	$\forall(i)$
13	$C_i^\star = Y_{ci}^\star$	$\forall(i)$
14	$\sum_{j=1}^{I} P_{mj}^\star \left(M_{cj}^\star + M_{mj}^\star + M_{xj}^\star\right) \pi_{ji} = P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
15	$X_i^\star = Y_{xi}^\star$	$\forall(i)$
16	$P_{ci}^\star = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_i^\star}{\alpha\nu_{ci}}\right)^{\alpha\nu_{ci}}\left(\frac{w_i^\star}{(1-\alpha)\nu_{ci}}\right)^{(1-\alpha)\nu_{ci}}\left(\frac{P_{mi}^\star}{1-\nu_{ci}}\right)^{1-\nu_{ci}}$	$\forall(i)$
17	$P_{mi}^\star = \gamma\left[\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i)$
18	$P_{xi}^\star = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_i^\star}{\alpha\nu_{xi}}\right)^{\alpha\nu_{xi}}\left(\frac{w_i^\star}{(1-\alpha)\nu_{xi}}\right)^{(1-\alpha)\nu_{xi}}\left(\frac{P_{mi}^\star}{1-\nu_{xi}}\right)^{1-\nu_{xi}}$	$\forall(i)$
19	$\pi_{ij}^\star = \frac{(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}{\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}$	$\forall(i,j)$
20	$B_i^\star = P_{mi}^\star\left(Y_{mi}^\star - M_i^\star\right) + q^\star \mathcal{A}_i$	$\forall(i)$
21	$P_{ci}^\star C_i^\star + P_{xi}^\star X_i^\star = r_i^\star K_i^\star + w_i^\star L_i^\star + q^\star \mathcal{A}_i$	$\forall(i)$
22	$X_i^\star = \delta K_i^\star$	$\forall(i)$
23	$r_i^\star = \left(\frac{\Phi_{1i}^\star}{\beta} + \Phi_{2i}^\star\right) P_{xi}^\star$	$\forall(i)$
24	$q^\star = 1/\beta - 1$	

Note: $u_{mj}^\star = \left(\frac{r_j^\star}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_j^\star}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mj}^\star}{1-\nu_m}\right)^{1-\nu_m}$.

B Derivations of structural relationships

This Appendix shows the derivations of key structural relationships. We refer to Table A.1 for the derivations and omit time subscripts to simplify notation. We begin by deriving an expression for $\frac{w_i}{P_{mi}}$ that will be used repeatedly.

Combining conditions 17 and 19, we obtain

$$\pi_{ii} = \gamma^{-\theta} \left(\frac{u_{mi}^{-\theta} T_{mi}}{P_{mi}^{-\theta}} \right).$$

Use the fact that $u_{mi} = B_{mi} r_i^{\alpha\nu_{mi}} w_i^{(1-\alpha)\nu_{mi}} P_{mi}^{1-\nu_{mi}}$, where B_{mi} is a collection of country-specific constants; then rearrange to obtain

$$P_{mi} = \left(\frac{T_{mi}}{\pi_{ii}} \right)^{-\frac{1}{\theta}} \left(\frac{r_i}{w_i} \right)^{\alpha\nu_{mi}} \left(\frac{w_i}{P_{mi}} \right)^{\nu_{mi}} P_{mi}$$

$$\Rightarrow \frac{w_i}{P_{mi}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\theta}}}{\gamma B_{mi}} \right)^{\frac{1}{\nu_{mi}}} \left(\frac{w_i}{r_i} \right)^{\alpha}. \tag{B.1}$$

Note that this relationship holds in both the steady state and along the transition.

Relative prices We show how to derive the price of consumption relative to intermediates; the relative price of investment is analogous. Begin with condition 16 to obtain

$$P_{ci} = \left(\frac{B_{ci}}{A_{ci}} \right) \left(\frac{r_i}{w_i} \right)^{\alpha\nu_{ci}} \left(\frac{w_i}{P_{mi}} \right)^{\nu_{ci}} P_{mi},$$

where B_{ci} is a collection of country-specific constants. Substitute equation (B.1) into the previous expression and rearrange to obtain

$$\frac{P_{ci}}{P_{mi}} = \left(\frac{B_{ci}}{A_{ci}} \right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}} \right)^{\frac{1}{\theta}}}{\gamma B_{mi}} \right)^{\frac{\nu_{ci}}{\nu_{mi}}}. \tag{B.2}$$

Analogously,

$$\frac{P_{xi}}{P_{mi}} = \left(\frac{B_{xi}}{A_{xi}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_{mi}}\right)^{\frac{\nu_{xi}}{\nu_{mi}}}. \tag{B.3}$$

Note that these relationships hold in both the steady state and along the transition.

Income per worker We define (real) income per worker in our model as

$$y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}.$$

We invoke conditions from Table A.1 for the remainder of this derivation. Conditions 1-6, 10, and 11 imply that

$$\begin{aligned} r_i K_i + w_i L_i &= \frac{w_i L_i}{1-\alpha} \\ \Rightarrow y_i &= \left(\frac{1}{1-\alpha}\right) \left(\frac{w_i}{P_{ci}}\right). \end{aligned}$$

To solve for $\frac{w_i}{P_{ci}}$, we use condition 16:

$$\begin{aligned} P_{ci} &= \frac{B_{ci}}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_{ci}} \left(\frac{w_i}{P_{mi}}\right)^{\nu_{ci}} P_{mi} \\ \Rightarrow \frac{P_{ci}}{w_i} &= \frac{B_{ci}}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_{ci}} \left(\frac{w_i}{P_{mi}}\right)^{\nu_{ci}-1}. \end{aligned}$$

Substituting equation (B.1) into the previous expression and exploiting the fact that $\frac{w_i}{r_i} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_i}{L_i}\right)$ yields

$$\begin{aligned} y_i &= \left(\frac{1}{1-\alpha}\right) \left(\frac{w_i}{P_{ci}}\right) \\ &= \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \left(\frac{A_{ci}}{B_{ci}}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{B_{mi}}\right)^{\frac{1-\nu_{ci}}{\theta\nu_{mi}}} \left(\frac{K_i}{L_i}\right)^{\alpha}. \end{aligned} \tag{B.4}$$

Steady-state capital-labor ratio and income We derive a structural relationship for the capital-labor ratio in the steady state only and refer to conditions in Table A.2.

Conditions 1-6 together with conditions 10 and 11 imply that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_i}{r_i}\right).$$

Using condition 23, we know that

$$r_i = \left(\frac{\Phi_1}{\beta} + \Phi_2\right) P_{xi},$$

which, by substituting into the prior expression, implies that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{(1-\alpha)\left(\frac{\Phi_1}{\beta} + \Phi_2\right)}\right)\left(\frac{w_i}{P_{xi}}\right),$$

which leaves the problem of solving for $\frac{w_i}{P_{xi}}$. Equations (B.1) and (B.3) imply

$$\begin{aligned}\frac{w_i}{P_{xi}} &= \left(\frac{w_i}{P_{mi}}\right)\left(\frac{P_{mi}}{P_{xi}}\right) \\ &= \left(\frac{A_{xi}}{B_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_{mi}}\right)^{\frac{1-\nu_{xi}}{\nu_{mi}}}\left(\frac{w_i}{r_i}\right)^{\alpha}.\end{aligned}$$

Substituting once more for $\frac{w_i}{r_i}$ in the previous expression yields

$$\left(\frac{w_i}{P_{xi}}\right)^{1-\alpha} = \left(\frac{\Phi_1}{\beta} + \Phi_2\right)^{-\alpha}\left(\frac{A_{xi}}{B_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_{mi}}\right)^{\frac{1-\nu_{xi}}{\nu_{mi}}}.$$

Solve for the aggregate capital-labor ratio

$$\frac{K_i}{L_i} = \left(\frac{\frac{\alpha}{1-\alpha}}{\left(\frac{\Phi_1}{\beta} + \Phi_2\right)^{-\frac{1}{1-\alpha}}}\right)\left(\frac{A_{xi}}{B_{xi}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_{mi}}\right)^{\frac{1-\nu_{xi}}{(1-\alpha)\nu_{mi}}}. \tag{B.5}$$

The steady-state income per worker, by invoking equation (B.5), can be expressed as

$$y_i = \left(\frac{\left(\frac{\Phi_1}{\beta} + \Phi_2\right)^{-\frac{\alpha}{1-\alpha}}}{1-\alpha} \right) \left(\frac{A_{ci}}{B_{ci}}\right) \left(\frac{A_{xi}}{B_{xi}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_{mi}} \right)^{\frac{1-\nu_{ci}+\frac{\alpha}{1-\alpha}(1-\nu_{xi})}{\nu_{mi}}} . \qquad \text{(B.6)}$$

Note that we invoked steady-state conditions, so this expression does not necessarily hold along the transition path.

C Data

This section describes the sources of data and any adjustments we make to the data to map it to the model.

The primary data sources include version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)), World Input-Output Database (Timmer, Dietzenbacher, Los, and de Vries, 2015; Timmer, Los, Stehrer, and de Vries, 2016, (WIOD)), 2011 World Bank's International Comparison Program (ICP), and Centre d'Etudes Prospectives et d'Informations Internationales (CEPII).[6]

Our data include 44 regions: 43 countries and a rest-of world aggregate (see Table C.1).

Production and trade We map the sectors in our model to the sectors in the data using two-digit categories in revision 3 of the International Standard Industrial Classification of All Economic Activities (ISIC). The intermediates correspond to categories 01-28; the investment sector corresponds to ISIC categories 29-35 and 45, respectively; and the consumption sector corresponds to the remaining categories.

Both value added and gross output for each of the three sectors are obtained directly from WIOD using the above classification.

We obtain bilateral trade data to trade in categories 01-28. Using the trade and production data, we construct bilateral trade shares for each country pair by following Bernard, Eaton, Jensen, and Kortum (2003) as follows:

$$\pi_{ij} = \frac{X_{ij}}{ABS_{bi}},$$

[6]ICP database can be found at http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html.

Table C.1: List of countries

Isocode	Country	Isocode	Country
AUS	Australia	IRL	Ireland
AUT	Austria	ITA	Italy
BEL	Belgium	JPN	Japan
BGR	Bulgaria	KOR	South Korea
BRA	Brazil	LTU	Lithuania
CAN	Canada	LUX	Luxembourg
CHE	Switzerland	LVA	Latvia
CHN	China	MEX	Mexico
CYP	Cyprus	MLT	Malta
CZE	Czech Republic	NLD	Netherlands
DEU	Germany	NOR	Norway
DNK	Denmark	POL	Poland
ESP	Spain	PRT	Potugal
EST	Estonia	ROU	Romania
FIN	Finland	RUS	Russia
FRA	France	SVK	Slovakia
GBR	United Kingdom	SVN	Slovenia
GRC	Greece	SWE	Sweden
HRV	Croatia	TUR	Turkey
HUN	Hungary	TWN	Taiwan
IDN	Indonesia	USA	United States
IND	India	ROW	Rest of World

where i denotes the importer, j denotes the exporter, X_{ij} denotes manufacturing trade flows from j to i, and ABS_i denotes country i's absorption defined as gross output less net exports of manufactures.

GDP, employment and prices We use data on output-side real GDP at current Purchasing Power Parity (2005 U.S. dollars) from PWT using the variable `cgdpo`. We convert this into U.S. dollars at market exchange rates by multiplying it by the price level of GDP at PPP, which is `pl_gdpo` in PWT. We use the variable `emp` from PWT 8.1 to measure the employment in each country. Our measure of real income is GDP at market exchange rates divided by the price level of consumption at PPP exchange rates, which is variable *pl_c* in the PWT, and corresponds to P_c in our model. The ratio $\frac{\texttt{cgdpo*pl\_gdpo}}{\texttt{pl\_cemp}}$ corresponds to GDP per worker, y, in our model.

The price of investment is obtained from PWT using variable `pl_i`. This corresponds to P_x in our model.

We construct the price of intermediate goods (manufactures) by combining disaggregate price data from the ICP. The data have several categories that fall under what we classify as manufactures: "Food and nonalcoholic beverages," "Alcoholic beverages, tobacco, and narcotics," "Clothing and foot wear," and "Machinery and equipment." The ICP reports expenditure data for these categories in both nominal U.S. dollars and real U.S. dollars. The PPP price equals the ratio of nominal expenditures to real expenditures. We compute the price level at PPP for manufactures as the sum of nominal expenditures across categories divided by the sum of real expenditures across categories. Nominal expenditures are measured in U.S. dollars at market exchange rates, while real expenditures are measured in real U.S. dollars at PPP exchange rates.

There is one more step before we take these prices to the model. The data correspond to expenditures and thus include additional margins such as distribution. To adjust for this, we first construct a price for distribution services. We assume that the price of distribution services is proportional to the overall price of services in each country and use the same method as above to compute the price across the following categories: "Housing, water, electricity, gas, and other fuels," "Health," "Transport," "Communication," "Recreation and culture," "Education," "Restaurants and hotels," and "Construction."

Now that we have the price of services in hand, we strip it away from the price of goods computed above to arrive at a measure of the price of manufactures that better maps to our model. In particular, let P_d denote the price of distribution services and P_g denote the price of goods that includes the distribution margin. We assume that $P_g = P_d^{\psi} P_m^{1-\psi}$, where P_m is the price of manufactures. We set $\psi = 0.45$, a value commonly used in the literature.

D Solution algorithm

In this Appendix, we describe the algorithm for computing (i) the initial steady state and (ii) the transition path. Before going further into the algorithms, we introduce some notation. We denote the steady-state objects using the $\star$ as a superscript; that is, $K_i^\star$ is the steady-state stock of capital in country i. We denote the vector of capital stocks across countries at time t as $\vec{K}_t = \{K_{it}\}_{i=1}^{I}$.

D.1 Computing the initial steady state

We use the technique from Mutreja, Ravikumar, and Sposi (2017), which builds on Alvarez and Lucas (2007), to solve for the steady state. The idea is to guess a vector of wages, then recover all remaining prices and quantities using optimality conditions and market clearing conditions, excluding the balance-of-payments condition. We then use departures from the balance-of-payments condition in each country to update our wage vector and iterate until we find a wage vector that satisfies the balance-of-payments condition. The following steps outline our procedure in more detail:

(i) We guess a vector of wages $\vec{w} \in \Delta = \{w \in \mathbb{R}^I_+ : \sum_{i=1}^{I} \frac{w_i L_i}{1-\alpha} = 1\}$; that is, with world GDP as the numéraire.

(ii) We compute prices $\vec{P}_c, \vec{P}_x, \vec{P}_m$, and $\vec{r}$ simultaneously using conditions 16, 17, 18, and 23 in Table A.2. The steady-state world interest rate is given condition 24. To complete this step, we compute the bilateral trade shares $\vec{\vec{\pi}}$ using condition 19.

(iii) We compute the aggregate capital stock as $K_i = \frac{\alpha}{1-\alpha} \frac{w_i L_i}{r_i}$, for all i, which derives easily from optimality conditions 1 and 4, 2 and 5, and 3 and 6, coupled with market clearing conditions for capital and labor 10 and11 in Table A.2.

(iv) We use condition 22 to solve for steady-state investment $\vec{X}$. Then we use condition 21 to solve for steady-state consumption $\vec{C}$.

(v) We combine conditions 4 and 13 to solve for $\vec{L}_c$, combine conditions 5 and 14 to solve for $\vec{L}_x$, and use condition 11 to solve for $\vec{L}_m$. Next we combine conditions 1 and 4 to solve for $\vec{K}_c$, combine conditions 2 and 5 to solve for $\vec{K}_M$, and combine conditions 3 and 6 to solve for $\vec{K}_x$. Similarly, we combine conditions 4 and 7 to solve for $\vec{M}_c$, combine conditions 5 and 8 to solve for $\vec{M}_m$, and combine conditions 6 and 9 to solve for $\vec{M}_x$.

(vi) We compute $\vec{Y}_c$ using condition 13, compute $\vec{Y}_m$ using condition 14, and compute $\vec{Y}_x$ using condition 15.

(vii) We compute an excess demand equation as in Alvarez and Lucas (2007) defined as

$$Z_i(\vec{w}) = \frac{P_{mi}Y_{mi} - P_{mi}M_i + q^{\star}\mathcal{A}_i}{w_i},$$

(the current account balance relative to the wage). Condition 20 requires that $Z_i(\vec{w}) = 0$ for all i. If the excess demand is sufficiently close to 0, then we have a steady state.

If not, we update the wage vector using the excess demand as follows:

$$\Lambda_i(\vec{w}) = w_i \left(1 + \psi \frac{Z_i(\vec{w})}{L_i}\right),$$

which is the updated wage vector, where ψ is chosen to be sufficiently small so that $\Lambda > 0$. Note that $\sum_{i=1}^{I} \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = \sum_{i=1}^{I} \frac{w_i L_i}{1-\alpha} + \psi \sum_{i=1}^{I} w_i Z_i(\vec{w})$. As in Alvarez and Lucas (2007), it is easy to show that $\sum_{i=1}^{I} w_i Z_i(\vec{w}) = 0$ which implies that $\sum_{i=1}^{I} \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = 1$, and hence, $\Lambda : \Delta \to \Delta$. We return to step (ii) with our updated wage vector and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to 0. In our computations we find that our preferred convergence metric,

$$\max_{i=1}^{I} \{|Z_i(\vec{w})|\},$$

converges roughly monotonically towards 0.

D.2 Computing the transition path

The solution procedure boils down to two iterations. First, we guess a set of nominal investment rates at each point in time for every country. Given these investment rates, we adapt the algorithm of Sposi (2012) and iterate on the wages and the world interest rate to pin down the endogenous trade imbalances. Then we go back and update the nominal investment rates that satisfy the Euler equation for the optimal rate of capital accumulation.

To begin, we take the initial capital stock, K_{i1}, and the initial NFA position, $\mathcal{A}_{i1}$, as given in each country.

(i) Guess a path for nominal investment rates $\{\vec{\rho}_t\}_{t=1}^{T}$ and terminal NFA, $\vec{\mathcal{A}}_{T+1}$.

(ii) Guess the entire path for wages $\{\vec{w}_t\}_{t=1}^{T}$ and the world interest rate $\{q_t\}_{t=2}^{T}$, such that $\sum_i \frac{w_{it}L_{it}}{1-\alpha} = 1\,(\forall t)$.

(iii) In period 1, set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1 \vec{L}}{\vec{K}_1}\right)$ since the initial stock of capital is predetermined. Compute prices P_{c1}, P_{x1}, and P_{m1} simultaneously using conditions 16, 17, and 18 in Table A.1. Solve for investment, X_1, using

$$X_{it} = \rho_{it} \frac{w_{it}L_{it} + r_{it}K_{it}}{P_{xit}},$$

and then solve for the next-period capital stock, K_2, using condition 22. Repeat this set of calculations for period 2, then for period 3, and continue all the way through period T. To complete this step, compute the bilateral trade shares $\{\vec{\pi}_t\}_{t=1}^T$ using condition 19.

(iv) Computing the path for consumption and bond purchases is slightly more involved. This requires solving the intertemporal problem of the household. This is done in three steps. First, we derive the lifetime budget constraint. Second, we derive the fraction of lifetime wealth allocated to consumption in each period. And third, we recover the sequences for bond purchases and the stock of NFAs.

Deriving the lifetime budget constraint To begin, (omitting country subscripts for now) use the representative household's period budget constraint in condition 20 and combine it with the NFA accumulation technology in condition 21 to get

$$\mathcal{A}_{t+1} = r_t K_t + w_t L_t - P_{ct} C_t - P_{xt} X_t + (1 + q_t)\mathcal{A}_t.$$

Iterate the period budget constraint forward through time and derive a lifetime budget constraint. At time $t = 1$, the NFA position, A_{i1}, is given. Next, compute the NFA position at time $t = 2$:

$$\mathcal{A}_2 = r_1 K_1 + w_1 L_1 - P_{c1} C_1 - P_{x1} X_1 + (1 + q_1)\mathcal{A}_1.$$

Similarly, compute the NFA position at time $t = 3$, but do it so that it is in terms of the initial NFA position.

$$\begin{aligned} \mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - P_{c2} C_2 - P_{x2} X_2 + (1 + q_2)\mathcal{A}_2 \\ \Rightarrow \mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - P_{x2} X_2 + (1 + q_2)(r_1 K_1 + w_1 L_1 - P_{x1} X_1) \\ &\quad - P_{c2} C_2 - (1 + q_2) P_{c1} C_1 + (1 + q_2)(1 + q_1)\mathcal{A}_{i1}. \end{aligned}$$

Continue to period 4 in a similar way:

$$
\begin{aligned}
&\mathcal{A}_4 = r_3K_3 + w_3L_3 - P_{c3}C_3 - P_{x3}X_3 + (1+q_3)\mathcal{A}_3 \\
\Rightarrow\ &\mathcal{A}_4 = r_3K_3 + w_3L_3 - P_{x3}X_3 \\
&\quad + (1+q_3)(r_2K_2 + w_2L_2 - P_{x2}X_2) \\
&\quad + (1+q_3)(1+q_2)(r_1K_1 + w_1L_1 - P_{x1}X_1) \\
&\quad - P_{c3}C_3 - (1+q_3)P_{c2}C_2 - (1+q_3)(1+q_2)P_{c1}C_1 + (1+q_3)(1+q_2)(1+q_1)\mathcal{A}_1.
\end{aligned}
$$

Before proceeding, it will be useful to define $(1+\mathcal{Q}_t) \equiv \prod_{n=1}^{t}(1+q_n)$:

$$
\begin{aligned}
\Rightarrow \mathcal{A}_4 = &\frac{(1+\mathcal{Q}_3)(r_3K_3 + w_3L_3 - P_{x3}X_3)}{(1+\mathcal{Q}_3)} \\
&+ \frac{(1+\mathcal{Q}_3)(r_2K_2 + w_2L_2 - P_{x2}X_2)}{(1+\mathcal{Q}_2)} \\
&+ \frac{(1+\mathcal{Q}_3)(r_1K_1 + w_1L_1 - P_{x1}X_1)}{(1+\mathcal{Q}_1)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c3}C_3}{(1+\mathcal{Q}_3)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c2}C_2}{(1+\mathcal{Q}_2)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c1}C_1}{(1+\mathcal{Q}_1)} \\
&+ (1+\mathcal{Q}_3)\mathcal{A}_1.
\end{aligned}
$$

By induction, for any time t,

$$
\begin{aligned}
&\mathcal{A}_{t+1} = \sum_{n=1}^{t}\frac{(1+\mathcal{Q}_t)(r_nK_n + w_nL_n - P_{xn}X_n)}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t}\frac{(1+\mathcal{Q}_t)P_{cn}C_n}{(1+\mathcal{Q}_n)} + (1+\mathcal{Q}_t)\mathcal{A}_1 \\
\Rightarrow\ &\mathcal{A}_{t+1} = (1+\mathcal{Q}_t)\left(\sum_{n=1}^{t}\frac{r_nK_n + w_nL_n - P_{xn}X_n}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t}\frac{P_{cn}C_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1\right).
\end{aligned}
$$

Finally, observe the previous expression as of $t = T$ and rearrange terms to derive the lifetime budget constraint:

$$\sum_{n=1}^{T} \frac{P_{cn} C_n}{(1+\mathcal{Q}_n)} = \underbrace{\sum_{n=1}^{T} \frac{r_n K_n + w_n L_n - P_{xn} X_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1 - \frac{\mathcal{A}_{T+1}}{(1+\mathcal{Q}_T)}}_{W}. \tag{D.1}$$

In the lifetime budget constraint (D.1), W denotes the net present value of lifetime wealth, taking both the initial and terminal NFA positions as given.

Solving for the path of consumption Next, compute how the net present value of lifetime wealth is optimally allocated over time. The Euler equation (condition 24) implies the following relationship between consumption in any two periods t and n:

$$C_n = \left(\frac{L_n}{L_t}\right) \beta^{\sigma(n-t)} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma} \left(\frac{P_{ct}}{P_{cn}}\right)^{\sigma} C_t$$

$$\Rightarrow \frac{P_{cn} C_n}{1+\mathcal{Q}_n} = \left(\frac{L_n}{L_t}\right) \beta^{\sigma(n-t)} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma-1} \left(\frac{P_{ct}}{P_{cn}}\right)^{\sigma-1} \frac{P_t C_t}{1+\mathcal{Q}_t}.$$

Since equation (D.1) implies that $\sum_{n=1}^{T} \frac{P_{cin} C_{in}}{1+\mathcal{Q}_n} = W$, rearrange the previous expression (putting country subscripts back in) to obtain

$$\frac{P_{cit} C_{it}}{1+\mathcal{Q}_{it}} = \underbrace{\left(\frac{L_{it} \beta^{\sigma t} (1+\mathcal{Q}_{it})^{\sigma-1} P_{cit}^{1-\sigma}}{\sum_{n=1}^{T} L_{in} \beta^{\sigma n} (1+\mathcal{Q}_{in})^{\sigma-1} P_{cin}^{1-\sigma}}\right)}_{\xi_{it}} W_i. \tag{D.2}$$

That is, each period the household spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T} \xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing bond purchases and the NFA positions In period 1, take as given consumption spending, investment spending, capital income, labor income, and net income from the initial NFA position to solve for net bond purchases $\{\vec{B}_t\}_{t=1}^{T}$ using the period budget constraint in condition 20. Solve for the NFA position in period 2 using condition 21. Then given income and spending in period 2, recover the net bond purchases in period 2 and compute the NFA position for period 3. Continue this process through all points in time.

Trade balance condition We impose that net exports equal the current account less net foreign income from asset holding. That is,

$$Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \frac{P_{mit}Y_{mit} - P_{mit}M_{it} - B_{it} + q_t\mathcal{A}_{it}}{w_{it}}.$$

Condition 25 requires that $Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = 0$ for all (i, t) in equilibrium. If this is different from 0 in some country at some point in time, update the wages as follows.

$$\Lambda_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = w_{it}\left(1 + \psi\frac{Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right)}{L_{it}}\right)$$

is the updated wages, where ψ is chosen to be sufficiently small so that $\Lambda^{w} > 0$.

Normalizing model units The next part of this step is updating the equilibrium world interest rate. Recall that the numéraire is world GDP at each point in time: $\sum_{i=1}^{I}(r_{it}K_{it} + w_{it}L_{it}) = 1$ $(\forall t)$. For an arbitrary sequence of $\{q_{t+1}\}_{t=1}^{T}$, this condition need not hold. As such, update the world interest rate as

$$1 + q_t = \frac{\sum_{i=1}^{I}(r_{it-1}K_{it-1} + \Lambda_{it-1}^{w}L_{it-1})}{\sum_{i=1}^{I}(r_{it}K_{it} + \Lambda_{it}^{w}L_{it})} \text{ for } t = 2, \ldots, T. \tag{D.3}$$

The capital and the rental rate are computed in step (ii), while the wages are the values Λ^{w} above. The world interest rate in the initial period, q_1, has no influence on the model other than scaling the initial NFA position q_1A_{i1}; that is, it is purely nominal. We set $q_1 = \frac{1-\beta}{\beta}$ (the interest rate that prevails in a steady state) and choose A_{i1} so that q_1A_{i1} matches the desired initial NFA position in current prices.

Having updated the wages and the world interest rate, return to step (ii) and perform each step again. Iterate through this procedure until the excess demand is sufficiently close to 0. In the computations, we find that our preferred convergence metric,

$$\max_{t=1}^{T}\left\{\max_{i=1}^{I}\left\{\left|Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right)\right|\right\}\right\},$$

converges roughly monotonically toward 0. This provides the solution to a "sub-equilibrium" for an exogenously specified nominal investment rate.

(v) The last step of the algorithm is to update the nominal investment rate and terminal

NFA condition. Until now, the Euler equation for investment in capital, condition 23, has not been used. We compute an "Euler equation residual" as

$$Z_{it}^r\left(\{\vec{\rho}_t\}_{t=1}^T\right) = \beta^\sigma \left(\frac{\frac{r_{it+1}}{P_{xit+1}} - \Phi_2(K_{it+2}, K_{it+1})}{\Phi_1(K_{it+1}, K_{it})}\right)^\sigma \left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^\sigma - \left(\frac{C_{it+1}}{C_{it}}\right). \tag{D.4}$$

Condition 23 requires that $Z_{it}^r\left(\{\vec{\rho}_t\}_{t=1}^T\right) = 0$ for all (i,t) in equilibrium. We update the nominal investment rates as

$$\Lambda_{it}^r\left(\{\vec{\rho}_t\}_{t=1}^T\right) = \rho_{it}\left(1 + \psi Z_{it}^r\left(\{\vec{\rho}_t\}_{t=1}^T\right)\right). \tag{D.5}$$

To update ρ_{iT}, we need to define $\Phi_2(K_{iT+2}, K_{iT+1})$, which is simply its steady-state value, $\Phi_2^\star = \delta - \frac{1}{\lambda}$, which serves as a boundary condition for the transition path of capital stocks.

Given the updated sequence of nominal investment rates, return to step (i) and repeat. Continue iterationing until $\max_{t=1}^T \left\{\max_{i=1}^I \left\{|Z_{it}^r\left(\{\vec{\rho}_t\}_{t=1}^T\right)|\right\}\right\}$ is sufficiently close to 0.

Since the steady state cannot be determined independently from the transition path, we need to update our guess for the terminal (steady state) NFA position $\mathcal{A}_{iT+1}$. In our first iteration, we do not know what the steady state value is so we set it equal to 0. Given that initial guess, that first iteration is going to deliver a sequence of NFA positions that, by the turnpike theorem will converge to its steady-state value at some time $t^\star < T$. After our first iteration, we take the NFA position at $t^\star$ and use it as the terminal condition for our second iteration. We choose $t^\star$ as $T \times \text{floor}\left(\frac{\text{iterations}}{1+\text{iterations}}\right)$. In our algorithm we use $T = 150$, so that in iteration 2, $t^\star = 100$. This way of updating the terminal NFA position ensures that the model settles down to its steady state before and through T.

Our algorithm takes advantage of excess demand equations for our updating rules, just as in the computation of static models (e.g., Alvarez and Lucas (2007)). One advantage of using excess demand iteration is that we do not need to compute gradients to choose step directions or step size, as in the case of nonlinear solvers such as the ones used by Eaton, Kortum, Neiman, and Romalis (2016) and Kehoe, Ruhl, and Steinberg (2016). This saves a tremendous amount of computational time, particularly as the number of countries or the number of time periods is increased.

E Non-uniform trade liberalization

Our previous counterfactuals considered uniform reductions in trade frictions across countries. In practice these trade frictions include policy-induced impediments to trade as well as frictions not directly influenced by policy, such as geography. Most trade liberalizations involve reducing the policy-induced impediment to trade and, since the relative importance of this component is heterogeneous across countries, these trade liberalizations are non-uniform. We consider a counterfactual trade liberalization in which we remove the policy-induced impediments to trade.

In order to isolate the policy component from the non-policy component in the trade frictions, we project the calibrated bilateral trade frictions onto symmetric gravity variables, including geographic distance, common border, common language, and common currency.

We estimate the following equation

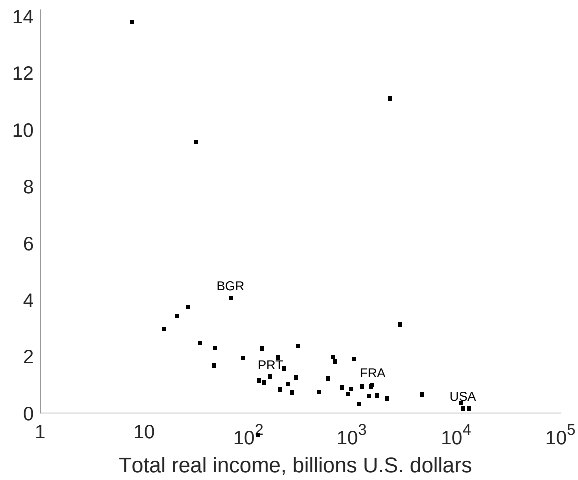
$$\log(d_{ij}) = \sum_{k=1}^{6} dist_{ij}^{k} + brdr_{ij} + lang_{ij} + curr_{ij} + e_j + \varepsilon_{ij}. \tag{E.1}$$

where $dist_{ij}^{k}$ is the contribution to trade costs of the distance between country j and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, $brdr_{ij}$, common language, $lang_{ij}$, and common currency, $curr_{ij}$. The term e_j is an exporter fixed effect, as in Waugh (2010).

Our assumption is that the impediments to trade that stem from these gravity variables cannot be altered by trade policy. The remainder of the trade frictions, which correspond to the exporter fixed effect and the residual, are asymmetric and could be affected by policy changes. We fist consider a policy that removes all asymmetries in trade frictions. We achieve this by: (i) setting the exporter fixed effect in each country equal to the minimum exporter fixed effect across countries (Germany, in our sample), and (ii) setting the residual for each country pair to the minimum value between the countries. For example, $\tilde{\varepsilon}_{ij} = \min(\varepsilon_{ij}, \varepsilon_{ji})$. Feature (ii) implies that after controlling for geography, there should be no difference between the cost of shipping from Cyprus to Germany and shipping from Germany to Cyprus. Given our estimates, we construct counterfactual trade frictions which we refer to as policy-free trade frictions.

The export-weighted trade frictions fall by 95 percent in Bulgaria, 76 percent in Portugal, 34 percent in France, and 40 percent in the United States. The elasticity of welfare gains associated with these reductions is 4.5 in Bulgaria, 1.7 in Portugal, 1.4 in France and 0.6 in the United States; see Figure E.1.

Figure E.1: Elasticity of gains from a nonuniform reduction in trade frictions



Notes: Elasticity of gain is computed as the absolute value of percent change in welfare divided by percent change in trade friction. Welfare gains are computed following the removal of non-gravity impediments to trade. The elasticity is computed as the absolute value of percent change in welfare divided by percent change in trade friction.

These elasticities imply that the scope for welfare gains through policy reform is greater for countries like Bulgaria than for countries like the United States.

F Model with heterogeneous input-output linkages

We enrich our baseline model by incorporating a complete input-output structure across four sectors. This builds on Caliendo and Parro (2015) where every sector's output goes into intermediate and final use. Different from Caliendo and Parro (2015), the final use is split into consumption and investment, thereby introducing dynamics via capital accumulation. We also introduce one-period bonds to allow for endogenous trade imbalances and current account dynamics.

Countries are indexed by $(i, j) = 1, \ldots, I$, sectors by $(n, k) = 1, \ldots, N$, and time by $t = 1, \ldots, T$. There are four sectors: durable and non-durable goods, and durable and non-durable services. In each sector, there is a continuum of varieties that are tradable. Trade in varieties is subject to iceberg costs. Each country has a representative household that owns the country's primary factors of production, capital and labor. Capital and labor are mobile across sectors within a country but are immobile across countries. The household inelastically supplies capital and labor to domestic firms, and purchases output from each sector and allocates it towards consumption and investments. Investment augments the stock of capital. Households can trade one-period bonds so that trade imbalances are endogenous. There is no uncertainty and households have perfect foresight.

Endowments The representative household in country i is endowed with workforce L_i, which is constant over time. Each period households supply labor inelastically. In period 1 the household in country i is endowed with an initial stock of capital, K_{i1}, and an initial net foreign asset (NFA) position, $\mathcal{A}_{i1}$. These stocks evolve endogenously throughout time based on investment and saving decisions.

Technology There is a unit interval of potentially tradable varieties in each sector indexed by $v^n \in [0,1]$ for $n = 1, \ldots, N$.

Within each sector, country i bundles all of the varieties with constant elasticity in order to construct a sectoral composite good according to

$$Q^n_{it} = \left[\int_0^1 Q^n_{it}(v^j)^{1-1/\eta} dv^n \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties. The term $Q^n_{it}(v^n)$ is the quantity of variety v^n used by country i at time t, which can be either imported or purchased domestically, to construct the sector n composite good. The composite good, Q^n_{it}, is allocated for domestic use as either an intermediate input or for final consumption or final investment.

Each variety can be produced using capital, labor, and composite goods. The technologies for producing each variety in each sector are given by

$$Y^n_{it}(v^n) = z^n_i(v^n) \left(A^n_i K^n_{it}(v^n)^{\alpha} L^n_{it}(v^n)^{1-\alpha} \right)^{\nu^n_i} \left(\prod_{k=1}^{N} M^{nk}_{it}(v^n)^{\mu^{nk}_i} \right)^{1-\nu^n_i}.$$

The term $M^{nk}_{it}(v^n)$ denotes the quantity of the composite good of type k used by country i to produce $y^n_{it}(v^n)$ units of variety v^n in sector n at time t. $K^n_{it}(v^n)$ denotes the amount of capital stock used and $L^n_{it}(v^n)$ denotes the amount of workers employed.

The country-specific parameter $\nu^n_i \in [0,1]$ is the share of value added in total output in sector n, while $\mu^{nk}_i \in [0,1]$ is the share of composite good k in total spending on intermediates by producers in sector n, with $\sum_k \mu^{nk}_i = 1$. The term α denotes capital's share in value added, which is constant across sectors and across countries. All of these parameters are constant over time.

The term A_i^n is the fundamental productivity, which scales value-added, of all varieties in sector n of country i. The term $z_i^n(v^n)$ scales gross-output of variety v^n in sector n of country i. Following Eaton and Kortum (2002), gross-output productivities in sector n for each variety are drawn independently from a Fréchet distribution with sector-specific shape parameter θ^n. The c.d.f. for idiosyncratic productivity draws in sector n in country i is $F^n(z) = \exp(-z^{-\theta^n})$.

Preferences The representative household's preferences are defined over consumption per worker, $\{C_{it}/L_i\}_{t=1}^T$:

$$\mathcal{U}_i = \sum_{t=1}^{T} \beta^{t-1} \frac{\left(\frac{C_{it}}{L_i}\right)^{1-1/\sigma}}{1-1/\sigma}.$$

Utility between adjacent periods is discounted by $\beta \in (0,1)$.

Consumption in country i at time t, C_{it}, bundles the consumption of composite goods from all sectors according to

$$C_{it} = \prod_{n=1}^{N} (C_{it}^n)^{\omega_i^{cn}},$$

where C_{it}^n denotes consumption of the sector n composite good by country i at time t and ω_i^{cn} denotes sector n's weight in the country i's consumption bundle (i.e., $\sum_{n=1}^N \omega_i^{cn} = 1$).

Capital accumulation Each period the representative household enters the period with K_{it} units of capital. A fraction δ depreciates during the period while investment, denoted by X_{it}, adds to the stock of capital subject to an adjustment cost. The stock of capital is then carried over into the next period. Thus, with $K_{i1} > 0$ given, the capital accumulation technology is

$$K_{it+1} = (1-\delta)K_{it} + \chi X_{it}^{\lambda} K_{it}^{1-\lambda}.$$

The term χ reflects the marginal efficiency of investment, and λ is the elasticity of capital accumulation with respect to investment.

Investment in country i at time t, X_{it}, bundles the investment of composite goods from all sectors according to

$$X_{it} = \prod_{n=1}^{N} (X_{it}^{n})^{\omega_i^{xn}}.$$

where X_{it}^{n} denotes investment of the sector n composite good by country i at time t and ω_i^{xn} denotes sector n's weight in the country i's investment bundle (i.e., $\sum_{n=1}^{N} \omega_i^{xn} = 1$).

Net-foreign asset accumulation Each period the representative household enters the period with a net-foreign asset (NFA) position $\mathcal{A}_{it}$. If $\mathcal{A}_{it} > 0$ then country i has, on net, a positive balance at time t, and a debt position otherwise. The NFA asset position is augmented by net purchases of one-period bonds, B_{it}, the current account balance. Thus, with $\mathcal{A}_{i1}$ given, the NFA position evolves according to

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + B_{it}.$$

Household constraints The household can borrow or lend to the rest of the world by trading one-period bonds, where B_{it} denotes the value of the net purchases of bonds. The world interest rate on one-period bonds at time t is denoted by q_t. If the household has a positive NFA position at time t, then net foreign income, $q_t\mathcal{A}_{it}$, is positive. Otherwise net foreign income is negative as resources go to pay off existing liabilities. The period budget constraint is given by

$$\sum_{n=1}^{N} (P_{it}^{n} C_{it}^{n} + P_{it}^{n} X_{it}^{n}) + B_{it} = r_{it} K_{it} + w_{it} L_i + q_t \mathcal{A}_{it}.$$

Consumption and investment in each sector must be non-negative.

Trade International trade is subject to barriers. Country i must purchase $d_{ij}^{n} \geq 1$ units of any variety of sector n from country j in order for one unit to arrive; $d_{ij}^{n} - 1$ units *melt* away in transit. As a normalization, $d_{ii}^{n} = 1$ for all (i, n).

Equilibrium A competitive equilibrium satisfies the following conditions: i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technologies for accumulating physical capital and assets, ii) taking prices as given, firms maximize profits subject to the available technologies, iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade barriers, and iv) markets clear. At each point in time, world GDP is defined as the numéraire: $\sum_{i=1}^{I} r_{it}K_{it} + w_{it}L_{it} = 1$. That is, all prices are expressed in units of current world GDP.

Calibration The calibration exercise is applied to 43 countries and a rest-of world aggregate. Economic activity is split across 4 sectors of the economy: (1) Durable goods; (2) Durable services; (3) Non-durable goods; (4) Non-durable services.

The primary data sources include version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)) and World Input-Output Database (Timmer, Dietzenbacher, Los, and de Vries, 2015; Timmer, Los, Stehrer, and de Vries, 2016, (WIOD)).

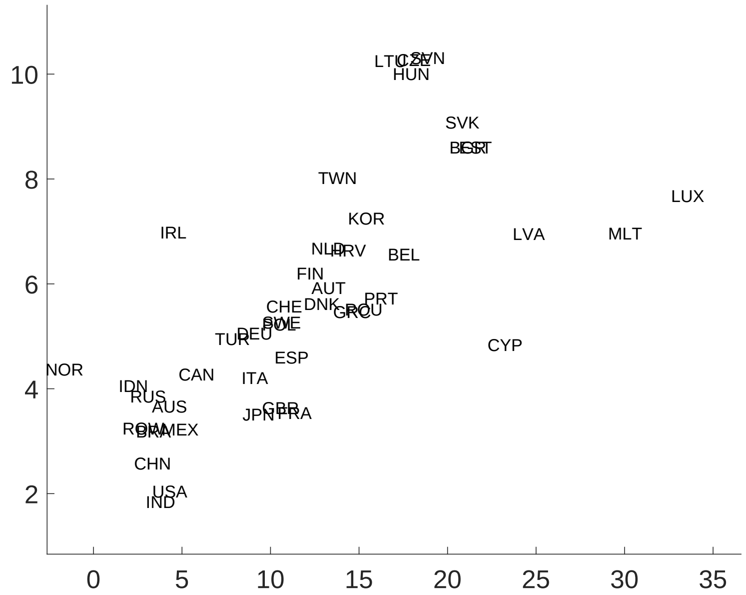
Our calibration uses data for 2014 and assumes that the world is in steady state in that year. This is the latest year for which both PWT and WIOD data are available.

We map sectors in our model to sector sin the data as follows. Non-durable goods sector corresponds to categories ISIC 01-28; durable goods sectors correspond to ISIC categories 29-35; durable services sector corresponds to ISIC 45; and non-durable services corresponds to the remaining ISIC categories.

Counterfactual We perform a uniform, permanent trade liberalization in which we reduce trade frictions of durable and non-durable goods sectors by 20 percent, respectively. We compute dynamic welfare gains from trade and compare the results to those in our baseline model (see Figure F.1). The vertical axis contains the gains in the full IO model. The horizontal axis contains the gains in the baseline model. We find that the gains are highly correlated in the two models, but tend to be lower in the full IO model.

To understand why the gains are lower in the full IO model, we compare changes in TFP and capital between steady states in the two models. Differences in the response of TFP are partly driven by the difference in the tradable-intensities of the consumption basket between the two models. In the baseline model, the average tradable intensity of the consumption basket is $1-\nu_c = 0.44$ and is $\omega^{c,DG}+\omega^{c,NG} = 0.23$ in the full IO model (G and DG correspond to durable goods and non-durable goods). A larger tradable-intensity in the baseline model contributes to a larger response of TFP in that model. Figure F.2a shows that countries that have a larger difference in this tradable intensity between the two models, also have a larger difference in the response of TFP. The steady-state change in TFP is defined as the ratio between the counterfactual and the initial steady states. Similarly, differences in the response of capital are partly driven by the difference in the tradable-intensities in the investment basket between the two models. In the baseline model, the average tradable intensity of the investment basket is $1 - \nu_x = 0.67$ and is $\omega^{x,DG} + \omega^{x,NG} = 0.29$ in the full

Figure F.1: Welfare gains from trade IO model and baseline model

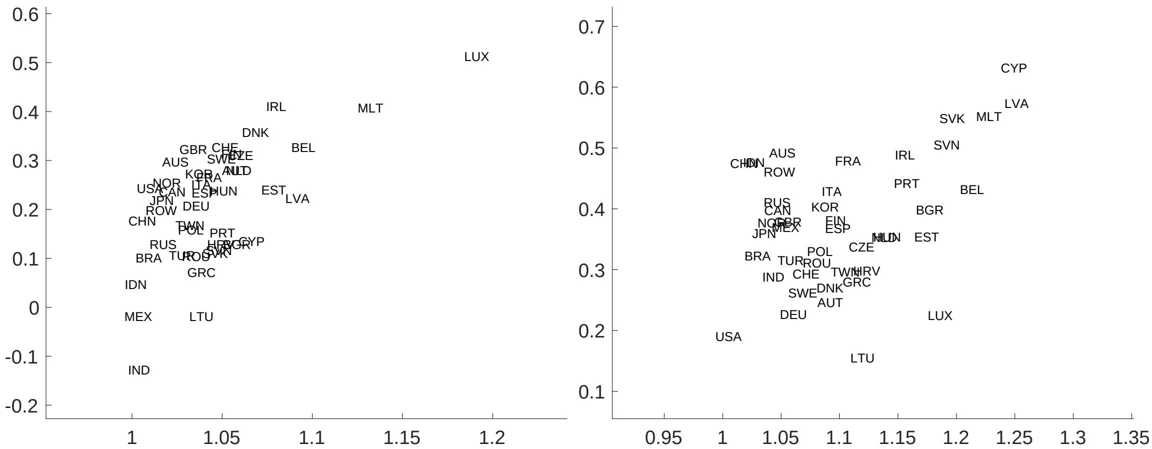


Notes: Results following a uniform, 20 percent liberalization. Horizontal axis-gains in the baseline model. Vertical axis-gains in the full IO model.

IO model. A larger tradable-intensity in the baseline model contributes to a larger response of capital in that model. Figure F.2b shows that countries that have a larger difference in this tradable intensity between the two models, also have a larger difference in the response of the capital stock. The steady-state change in the capital stock is defined as the ratio between the counterfactual and the initial steady states.

Figure F.2: TFP and capital component versus differences in tradable intensity (IO model and baseline model)

(a) TFP component (b) Capital component



Notes: Results following a uniform, 20 percent liberalization. Horizontal axis (a)-difference in tradable intensity in consumption in the baseline model minus that in the full IO model. Vertical axis (a)-the steady-state change in TFP in the baseline model relative to that in the full IO model. Horizontal axis (b)-difference in tradable intensity in investment in the baseline model minus that in the full IO model. Vertical axis (b)-the steady-state change in capital stock in the baseline model relative to that in the full IO model.